

## Chapter 3

# Memory

These slides support chapter 3 of the book

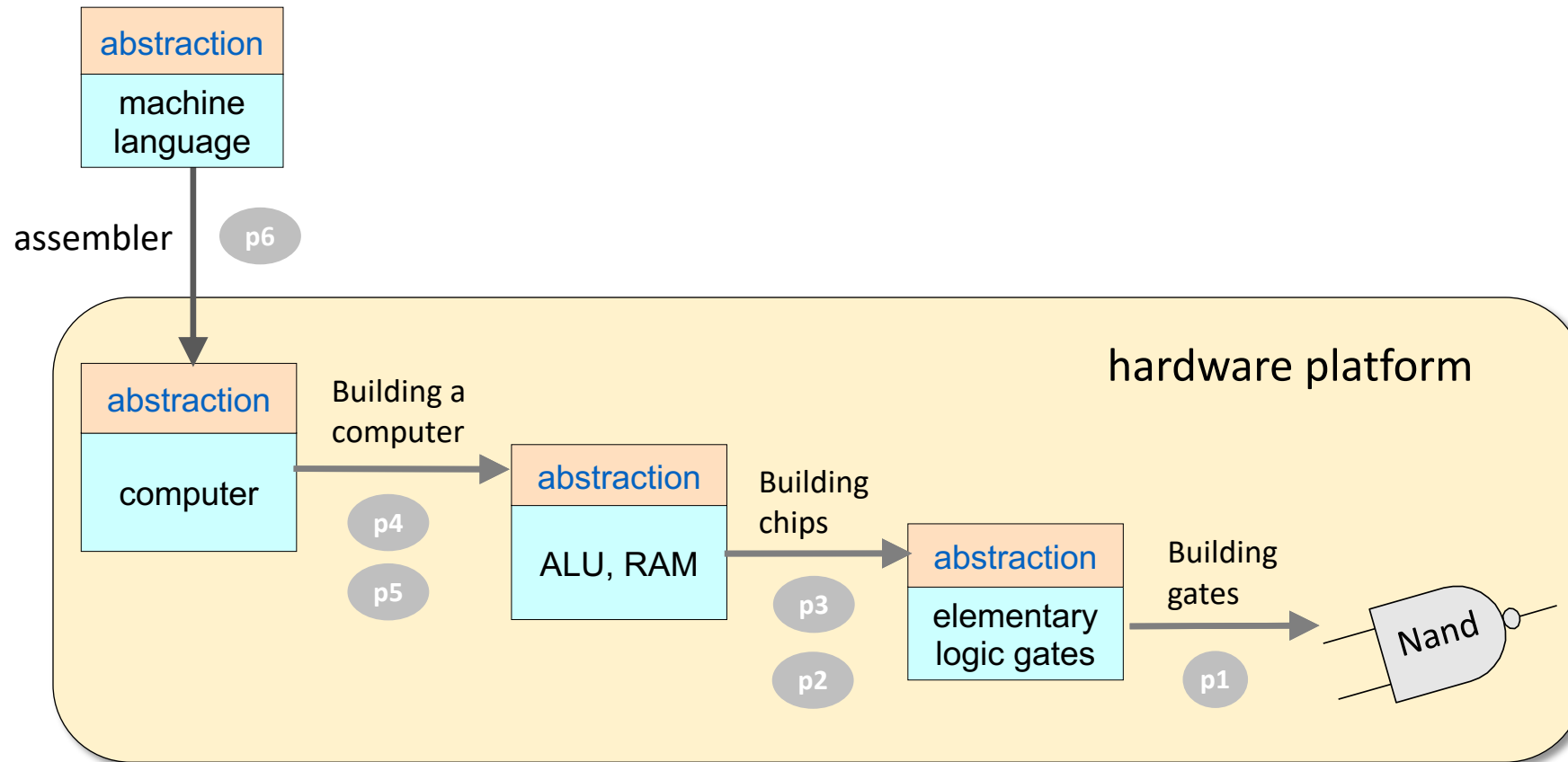
*The Elements of Computing Systems*

(1<sup>st</sup> and 2<sup>nd</sup> editions)

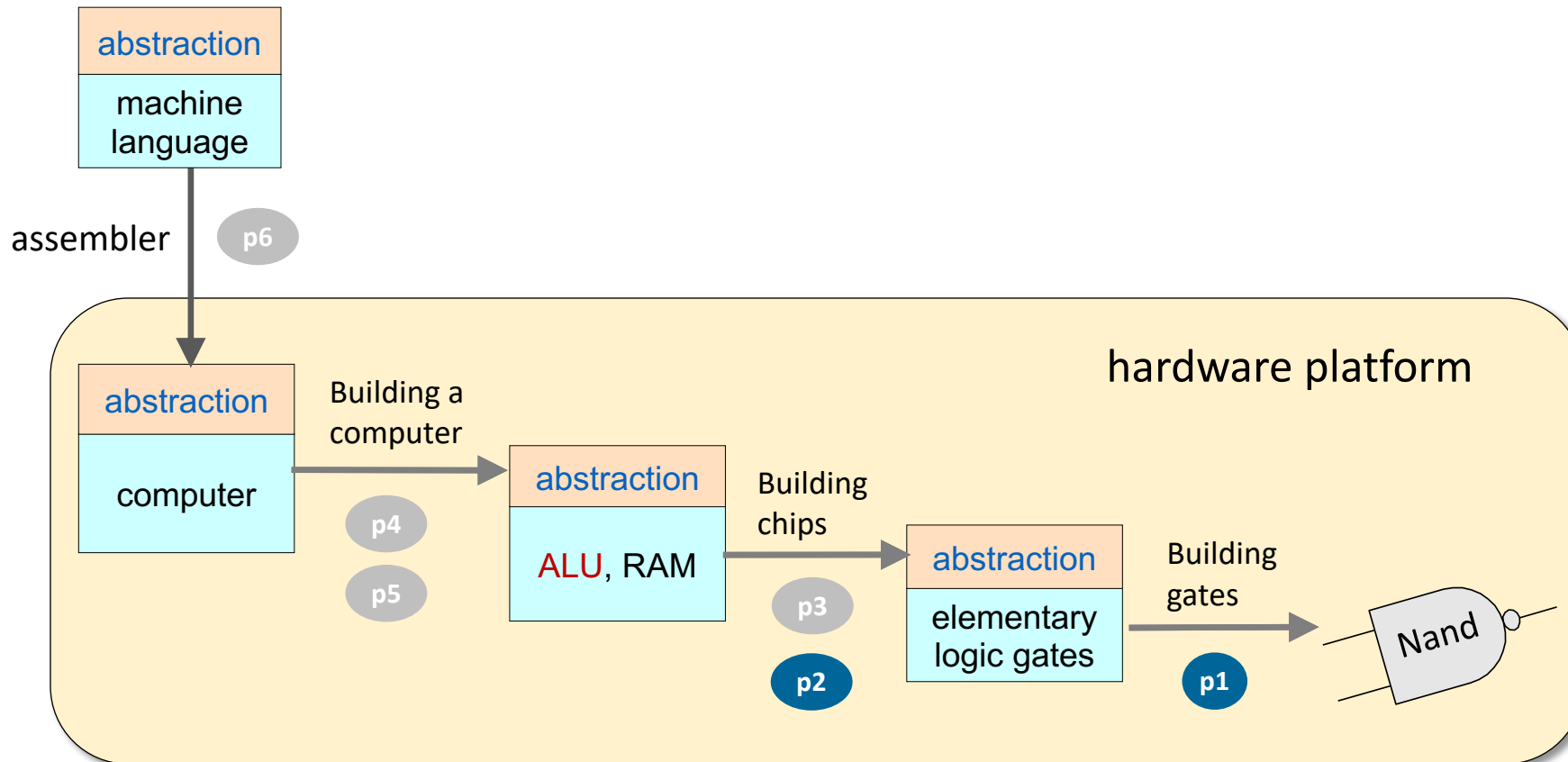
By Noam Nisan and Shimon Schocken

MIT Press

# Nand to Tetris Roadmap: Hardware



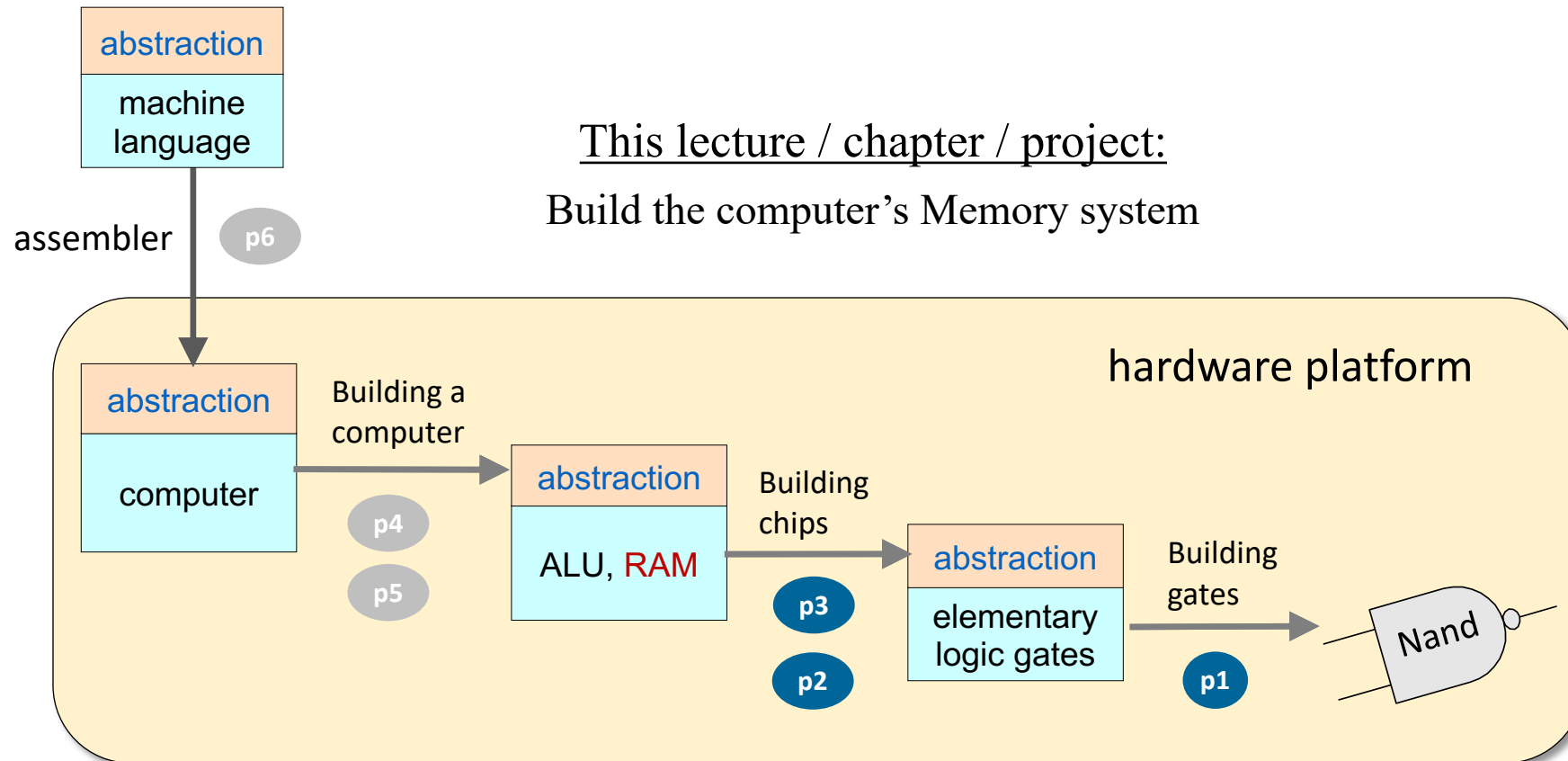
# Nand to Tetris Roadmap: Hardware



Project 1: Build basic logic gates

Project 2: Build the ALU

# Nand to Tetris Roadmap: Hardware



Project 1: Build basic logic gates

Project 2: Build the ALU

# A common theme in computer science

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- We present a simple model (the simpler, the better)
- We explore the model's power:
  - What the model can do
  - What it cannot do
- We then extend the model, to make it more powerful

Case in point:

Logic gates.

# Logic gates

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## Model: And, Or, Not, ...

- Simple, and powerful:  
Logic gates can realize any Boolean function, and can be combined to form powerful chips, like an ALU
- But, as a *general model of computation*, logic gates fall short

## Limitations

- Logic gates cannot store information (bits) over time
- Feedback loops are not allowed: A chip's output cannot serve as its input
- Logic gates can handle only inputs of a fixed size.  
For example, we can build an Or3 gate, and an Or4 gate, and so on, but we cannot build a single gate that computes Or for any given number of inputs

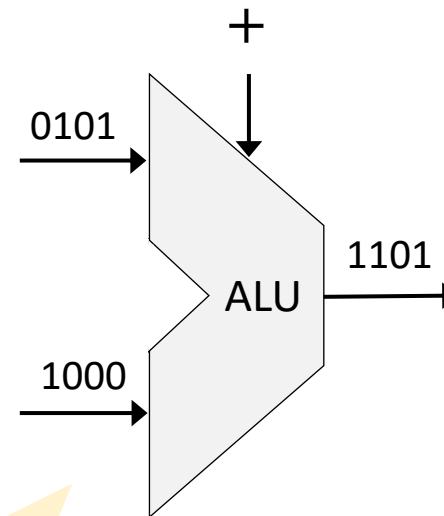
## Extension

Allow logic gates to be sensitive to the progression of *time*.

# Time-independent logic

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- So far we ignored *time*
- The chip's inputs were just “sitting there” – fixed and unchanging
- The chip's output was a function (“combination”) of the current inputs, and the current inputs only
- This style of gate logic is sometimes called:
  - *time-independent logic*
  - *combinational logic*
- All the chips that we discussed and developed so far were combinational



ALU: The “topmost”  
combinational chip

# Hello, time

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## Software needs:

- The hardware must be able to remember things, over time:
- The hardware must be able to do things, one at a time (sequentially):

Example (variables):

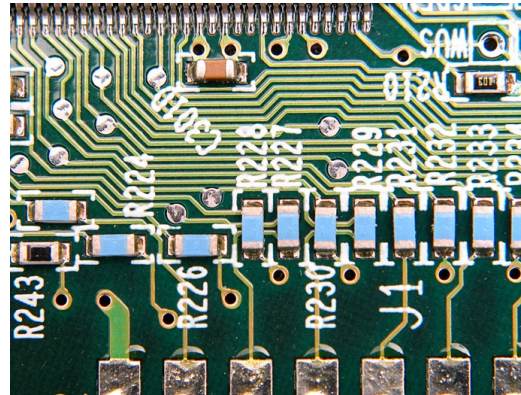
```
x = 17
```

Example (iteration):

```
for i in range(0, 10):  
    print(i)
```

## Hardware needs:

- The hardware must handle the *physical time delays* associated with *computing* and *moving* data from one chip to another.





# Hello, time

---

## Software needs:

- The hardware must be able to remember things, over time:
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x = 17
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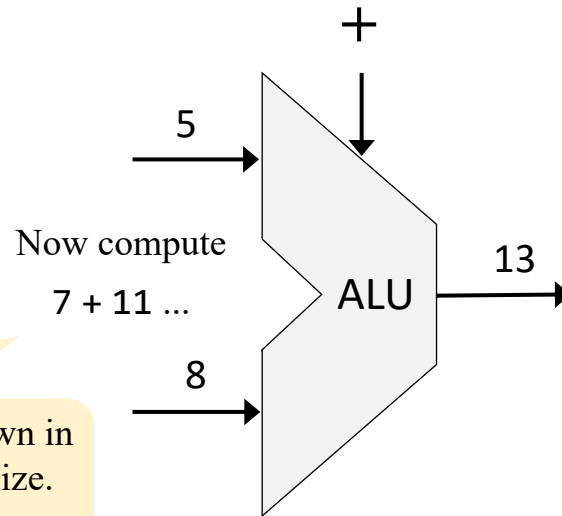
Example (iteration):

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## Hardware needs:

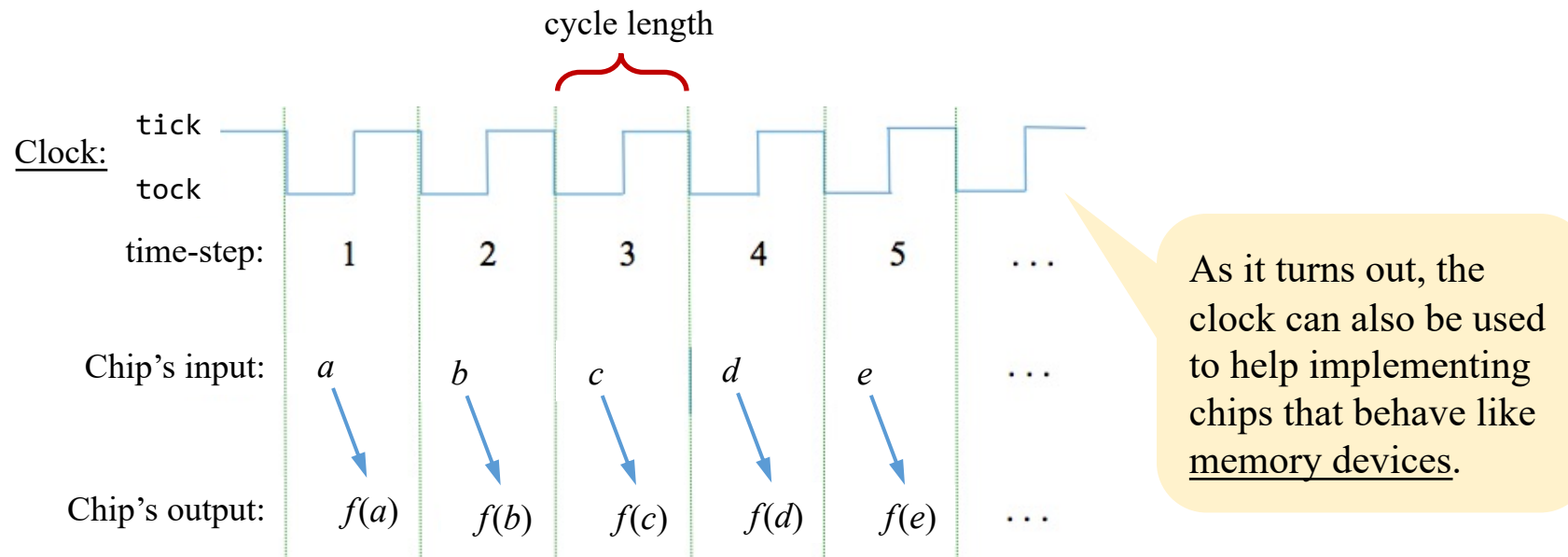
- The hardware must handle the *physical time delays* associated with *computing* and *moving* data from one chip to another.

It will take some time before 7 and 11 will settle down in the input ports, and before the sum  $7 + 11$  will stabilize. Till then, the ALU will output nonsense.



# Hello, time

Solution: We can neutralize the time delays if we decide to use *discrete time*



- Set the *cycle length* to be slightly  $>$  than the maximum time delay, and...
- Decide to use the chips' outputs only at the end of cycles (time-steps), ignoring what happens within cycles
- Details later.

# Memory

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**Memory:** The faculty of the brain by which data or information is encoded, stored, and retrieved when needed.

It is the *retention of information over time* for the purpose of influencing future action (Wikipedia)

Memory is time-based:

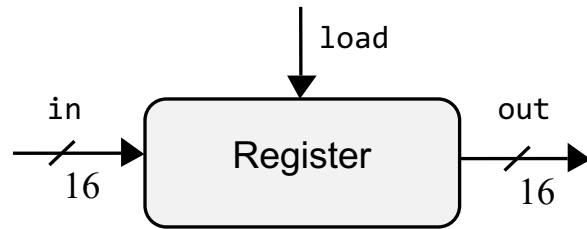
We remember *now* what was committed to memory *earlier*.



*It's a poor sort of memory  
that only works backwards.*

*-Lewis Carroll, through the White Queen*

# Memory



## Basic abstractions:

- “Loading” a value
- “Storing” a value



loading

storing

$x = 21, 21, 21, 21, 21, 21, 21, \dots$

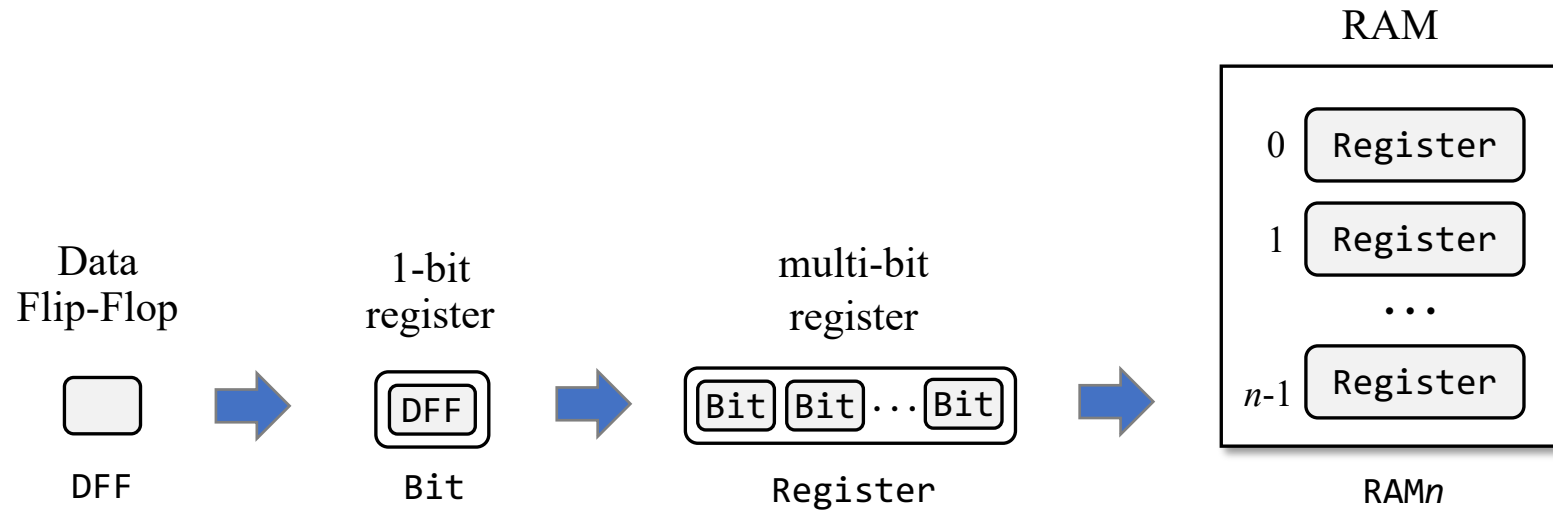
loading

storing

The challenge: Building chips that realize this functionality.

# Memory

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The challenge: Building chips that realize this functionality.

# Chapter 3: Memory

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## Abstraction

- Representing time
- Clock
- Registers
- RAM
- Counters

## Implementation

- Data Flip Flop
- Registers
- RAM
- Project 3: Chips
- Project 3: Guidelines

# Chapter 3: Memory

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## Abstraction

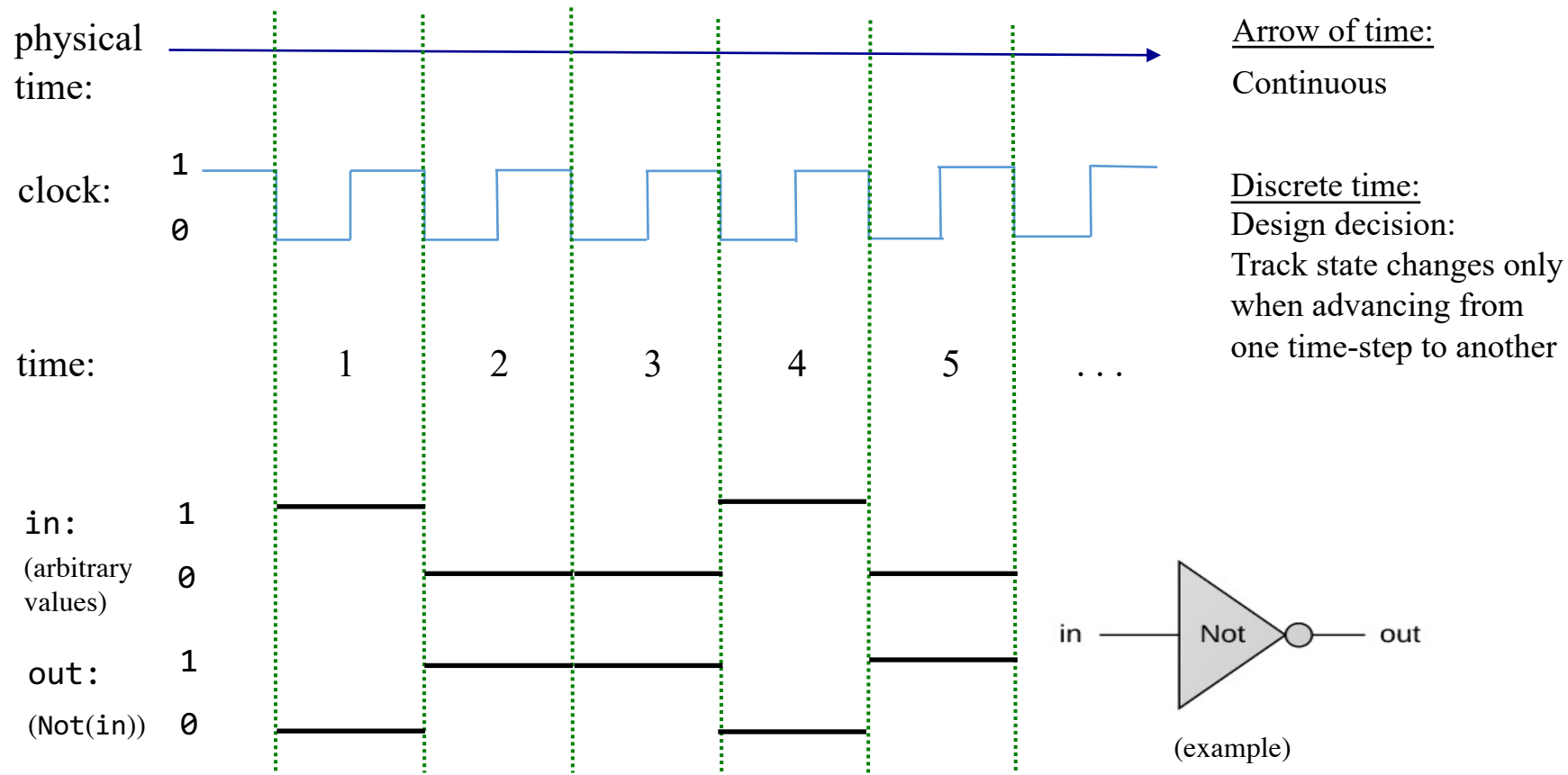
 Representing time

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# Chip behavior over time (example: Not gate)

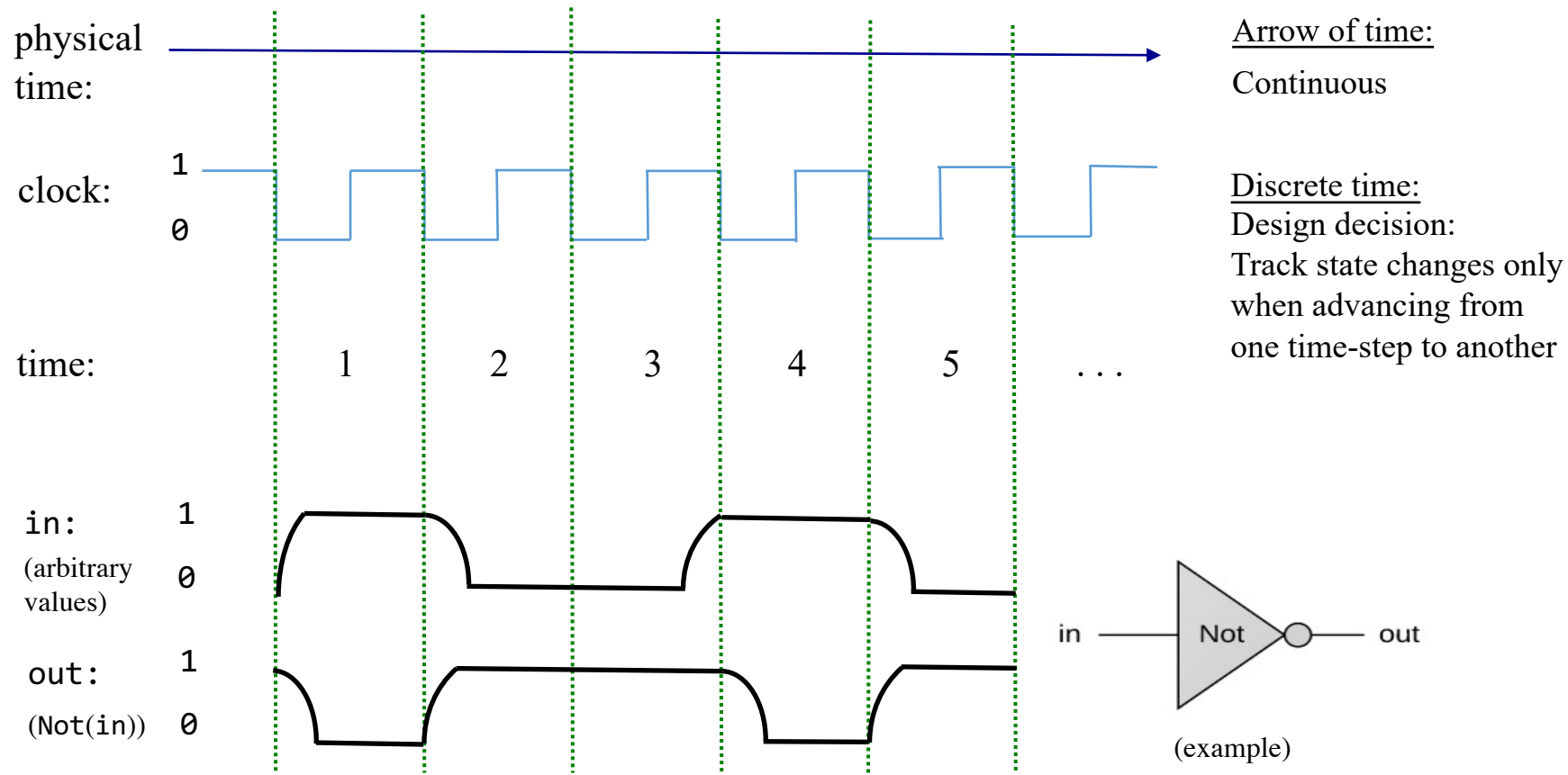


Desired / idealized behavior of the in and out signals:

That's how we *want* the hardware to behave

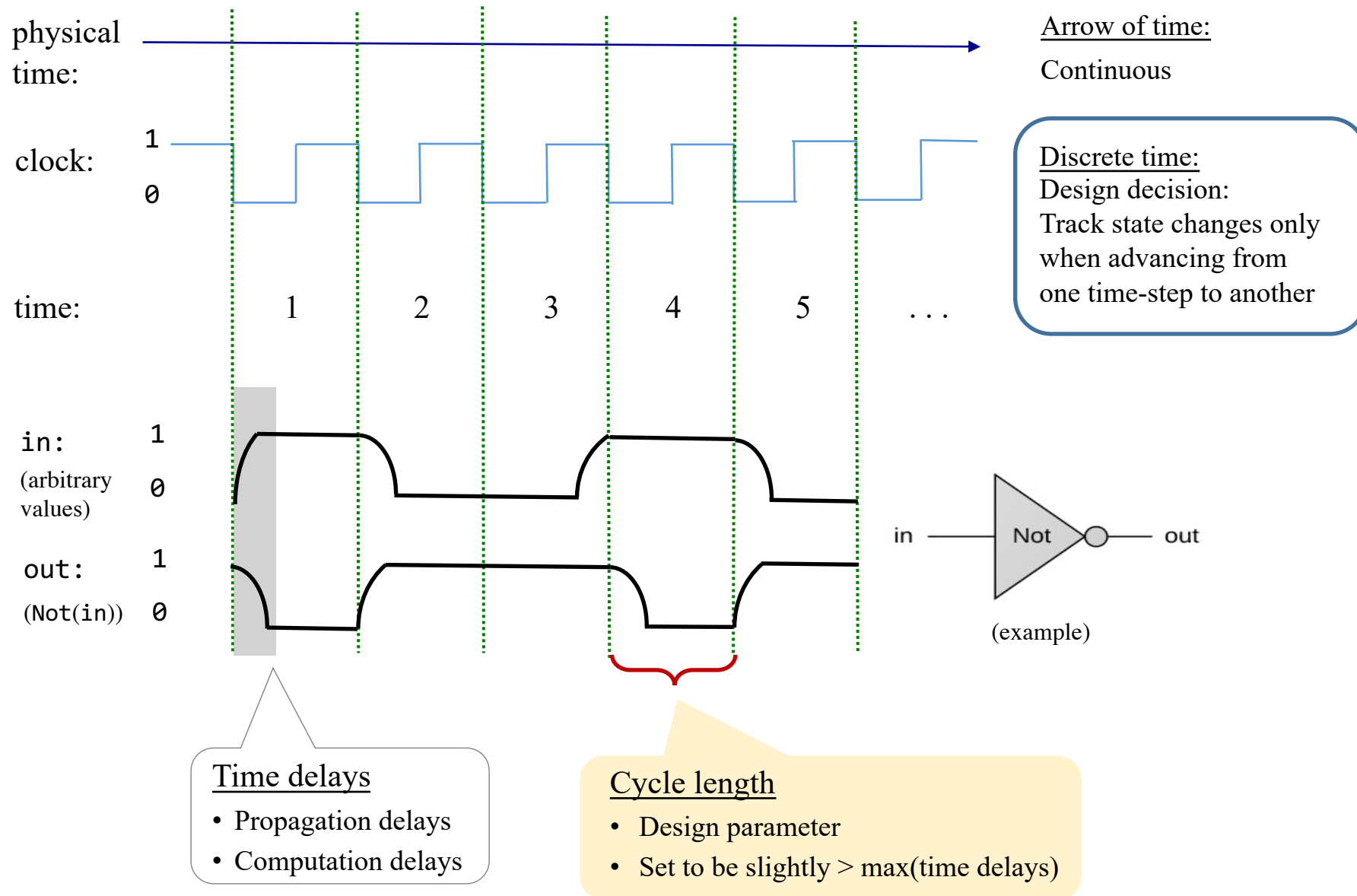


# Chip behavior over time (example: Not gate)

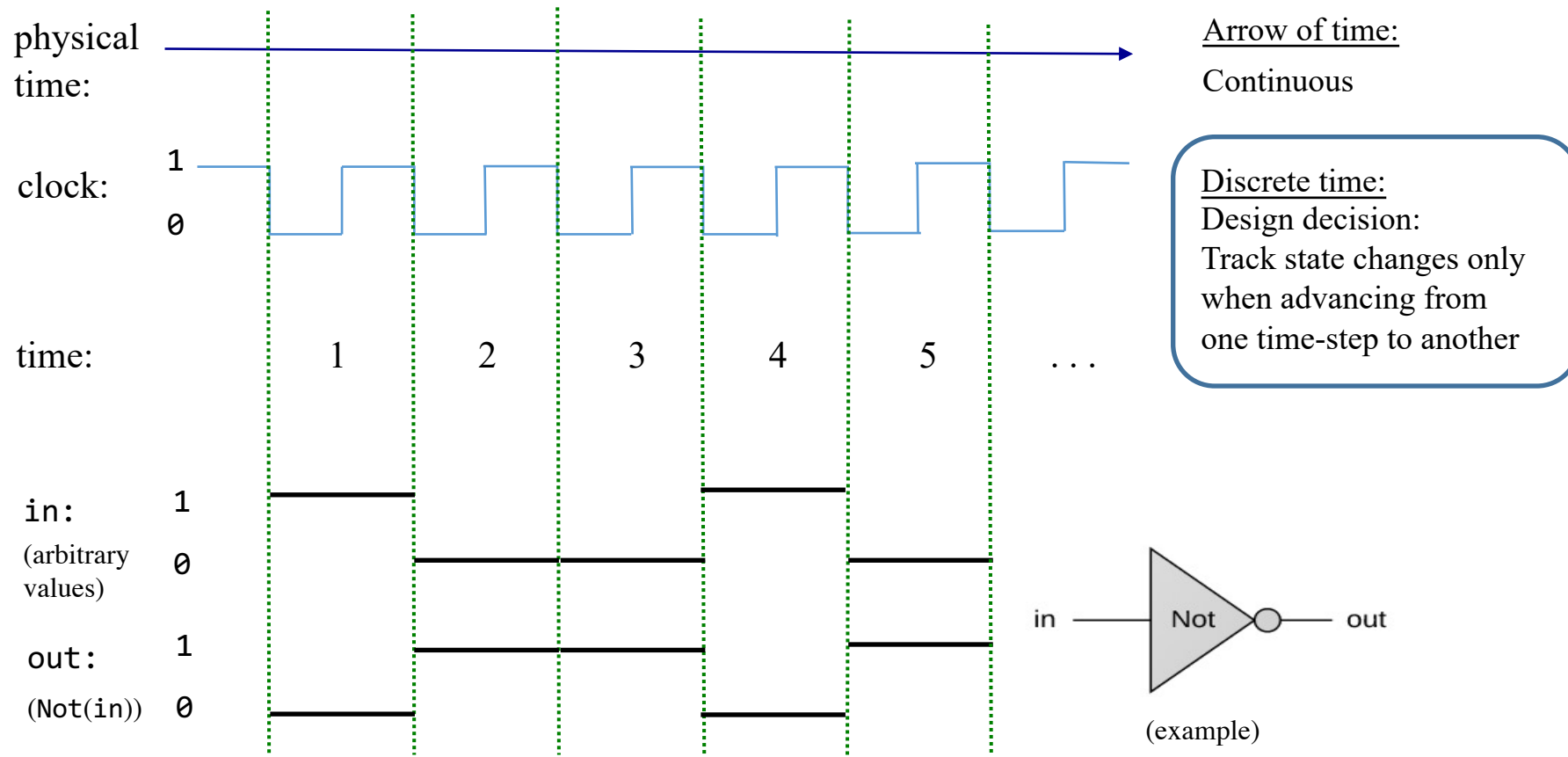


Actual behavior of the in and out signals:  
Influenced by physical time delays

# Chip behavior over time (example: Not gate)



# Chip behavior over time (example: Not gate)



## Resulting effect:

- Combinational chips react “immediately” to their inputs
- Facilitated by the decision to track changes only at cycle ends

# Chapter 3: Memory

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## Abstraction

✓ Representing time

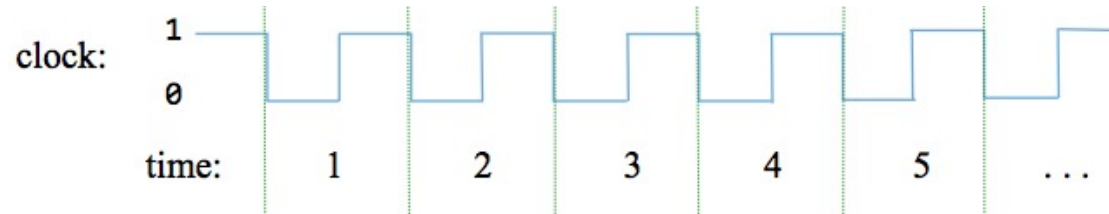
➡ Clock

- Registers
- RAM
- Counters

## Implementation

- Data Flip Flop
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- Project 3: Guidelines

# Clock: Simulated implementation



## Interactive simulation

A clock icon can be used to generate a sequence of tick-tock signals:

0, 0+, 1, 1+, 2, 2+, 3, 3+, ...

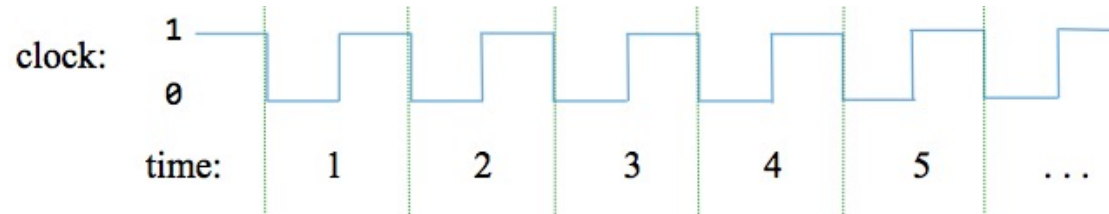


## Script-based simulation

“tick” and “tock” commands  
can be used to advance the clock:

```
...  
// Sets inputs, advances the clock, and  
// writes output values as it goes along.  
set in 19,  
set load 1,  
tick,  
output,  
tock,  
output,  
tick, tock,  
output,  
...
```

# Clock: Physical implementation



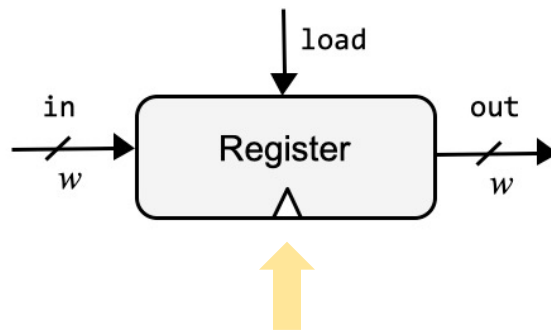
## Physical clock

- An *oscillator* is used to deliver an ongoing train of “tick/tock” signals



“1 MHz electronic oscillator circuit which uses the resonant properties of an internal quartz crystal to control the frequency. Provides the clock signal for digital devices such as computers.” (Wikipedia)

- The oscillator’s output is connected to all the time-based (clocked) chips in the computer



Chip diagram convention:  
A triangle icon represents a clock signal input

# Chapter 3: Memory

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## Abstraction

✓ Representing time

✓ Clock

➡ Registers

- RAM
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## Implementation

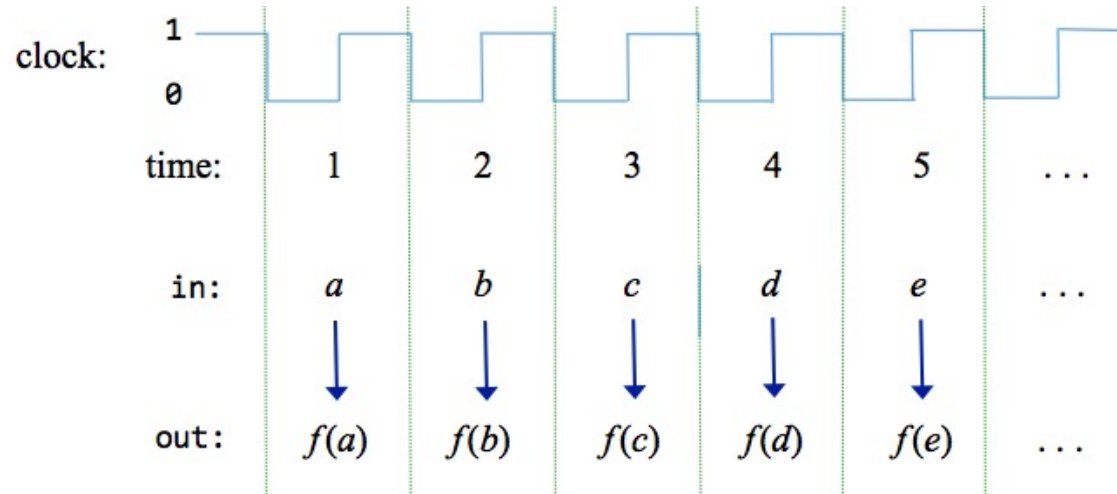
- Data Flip Flop
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# Combinational logic / Sequential logic

## Combinational logic:

The output depends on the current inputs

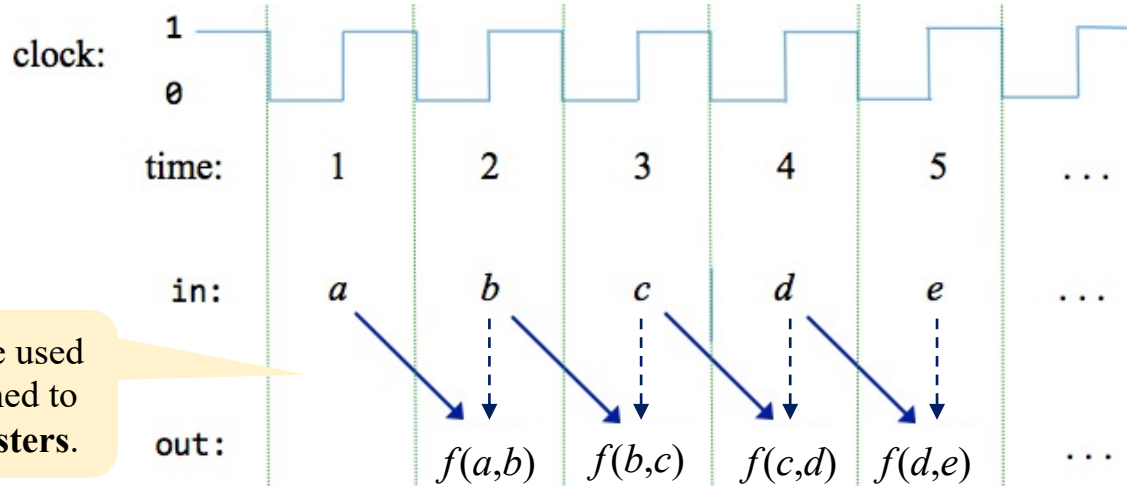
The clock is used to stabilize outputs



## Sequential logic:

The output depends on:

- Previous inputs
- Current inputs (optionally)

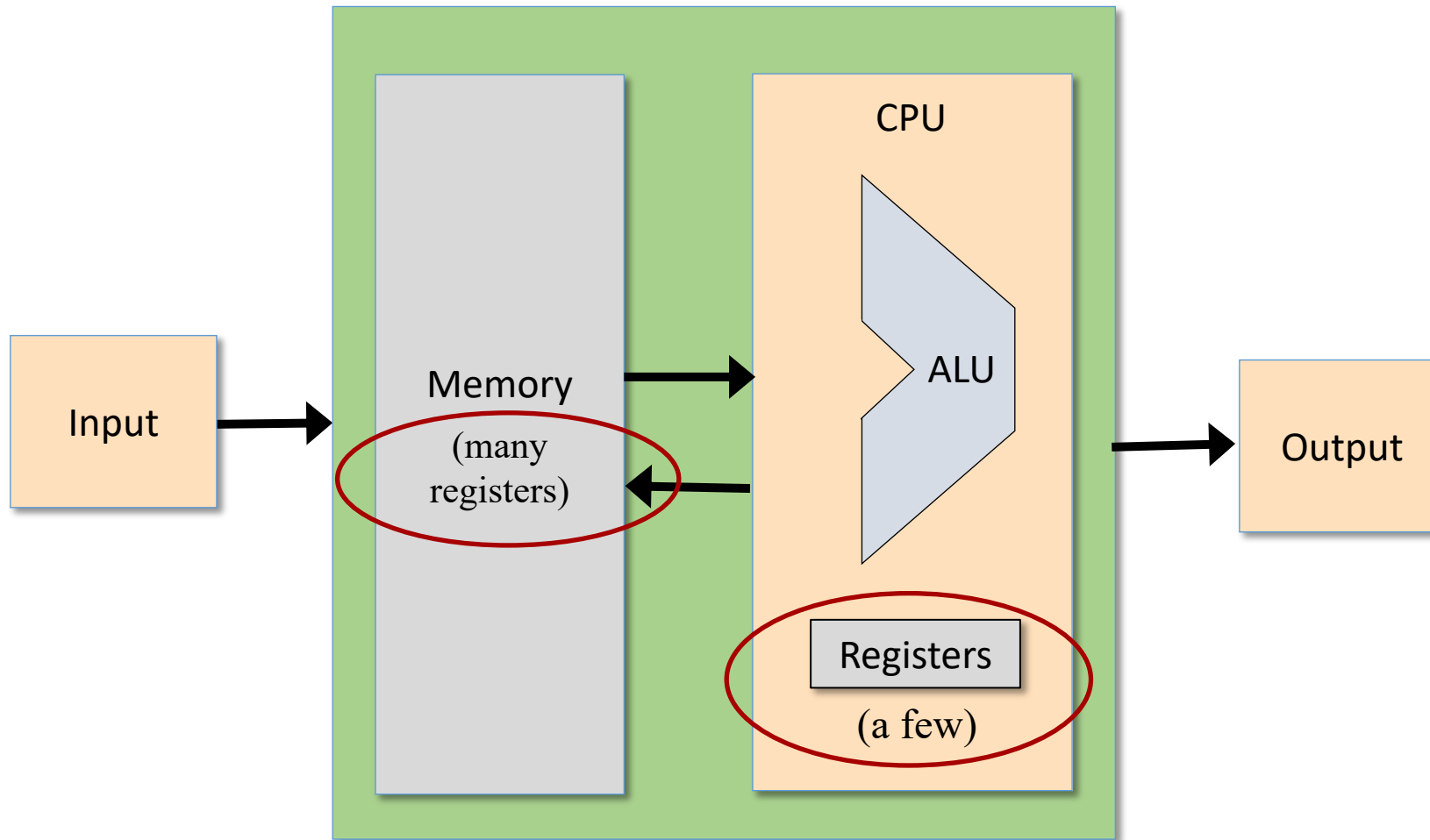


This behavior can be used to build chips designed to maintain state: **Registers**.



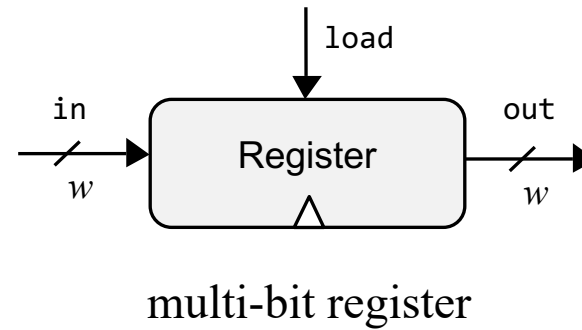
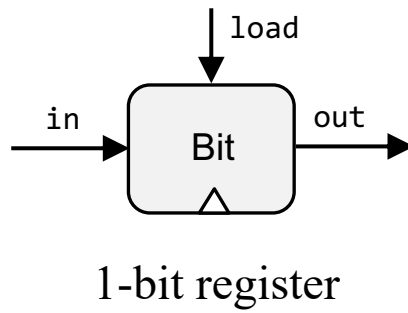
# Registers

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Computer Architecture

# Registers



## Designed to:

- “Store” / “remember” / “maintain” / “persist” a value , until...
- “Instructed” to “load”, and then “store”, another value.

time:

$x = 17, 17, 17, 17, 17, 17, 17, \dots, 17$

loading

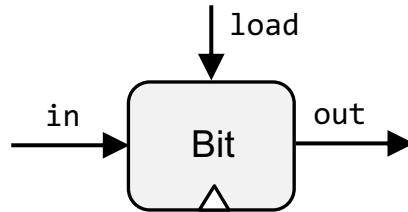
maintaining state

$x = 21, 21, 21, 21, 21, 21, \dots, 21$

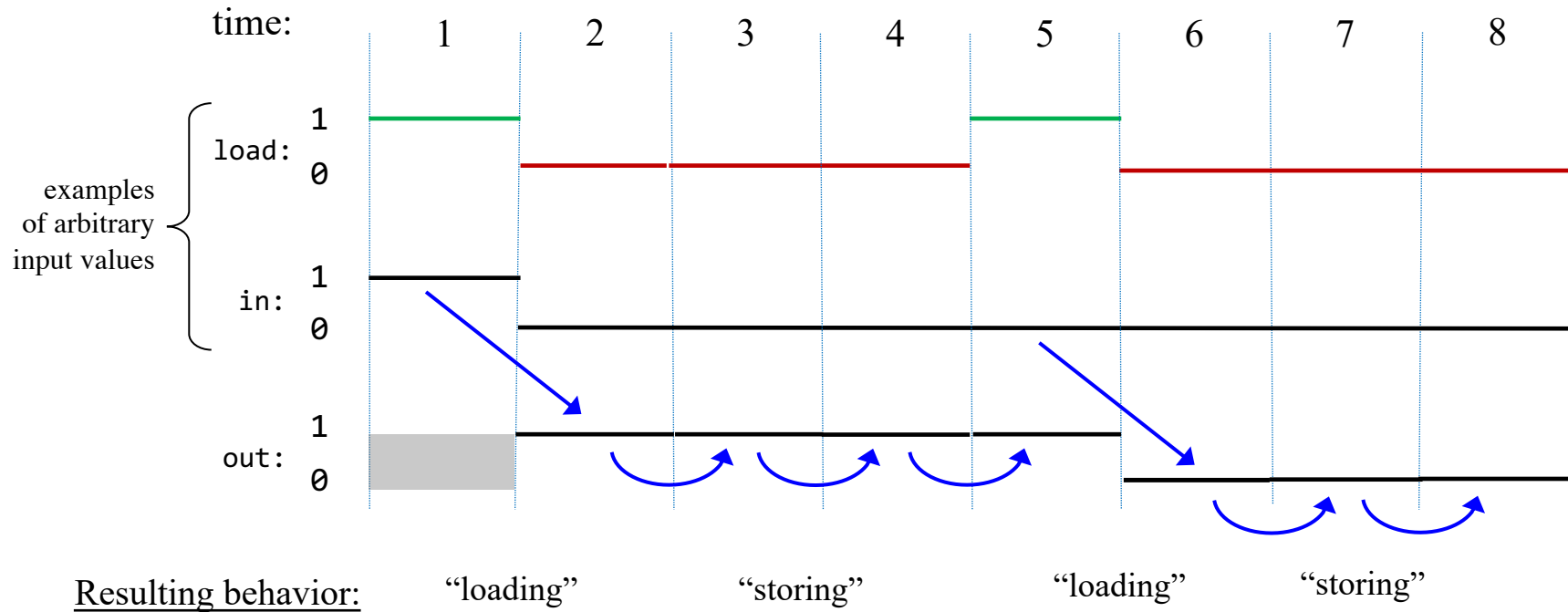
loading

maintaining state

# 1-Bit Register

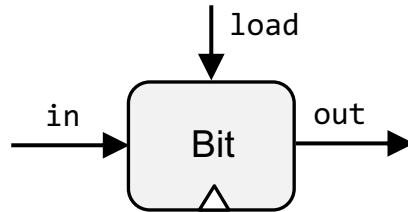


if  $\text{load}(t-1)$  then  $\text{out}(t) = \text{in}(t-1)$   
else  $\text{out}(t) = \text{out}(t-1)$



# 1-Bit Register

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if  $\text{load}(t-1)$  then  $\text{out}(t) = \text{in}(t-1)$   
else  $\text{out}(t) = \text{out}(t-1)$

## Usage:

### To read:

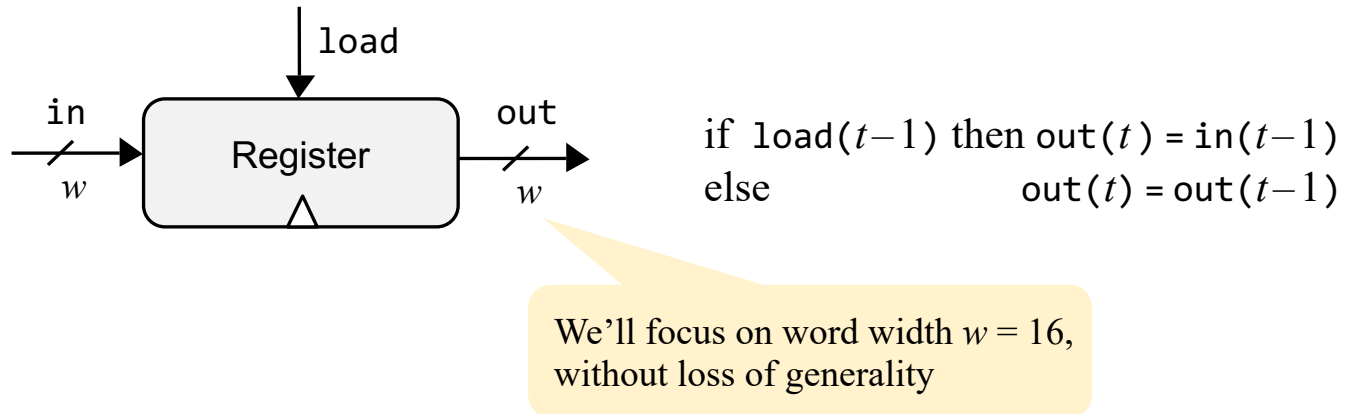
probe out      (out always emits the register's state)

### To write:

set  $\text{in} = v$       Result: The register's state becomes  $v$ ;  
set  $\text{load} = 1$       From the next time-step onward, out will emit  $v$

# Multi-bit Register

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Load / store behavior: Exactly the same as a 1-bit register

Read / write usage: Exactly the same as a 1-bit register



# Chapter 3: Memory

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## Abstraction

✓ Representing time

✓ Clock

✓ Registers

➡ RAM

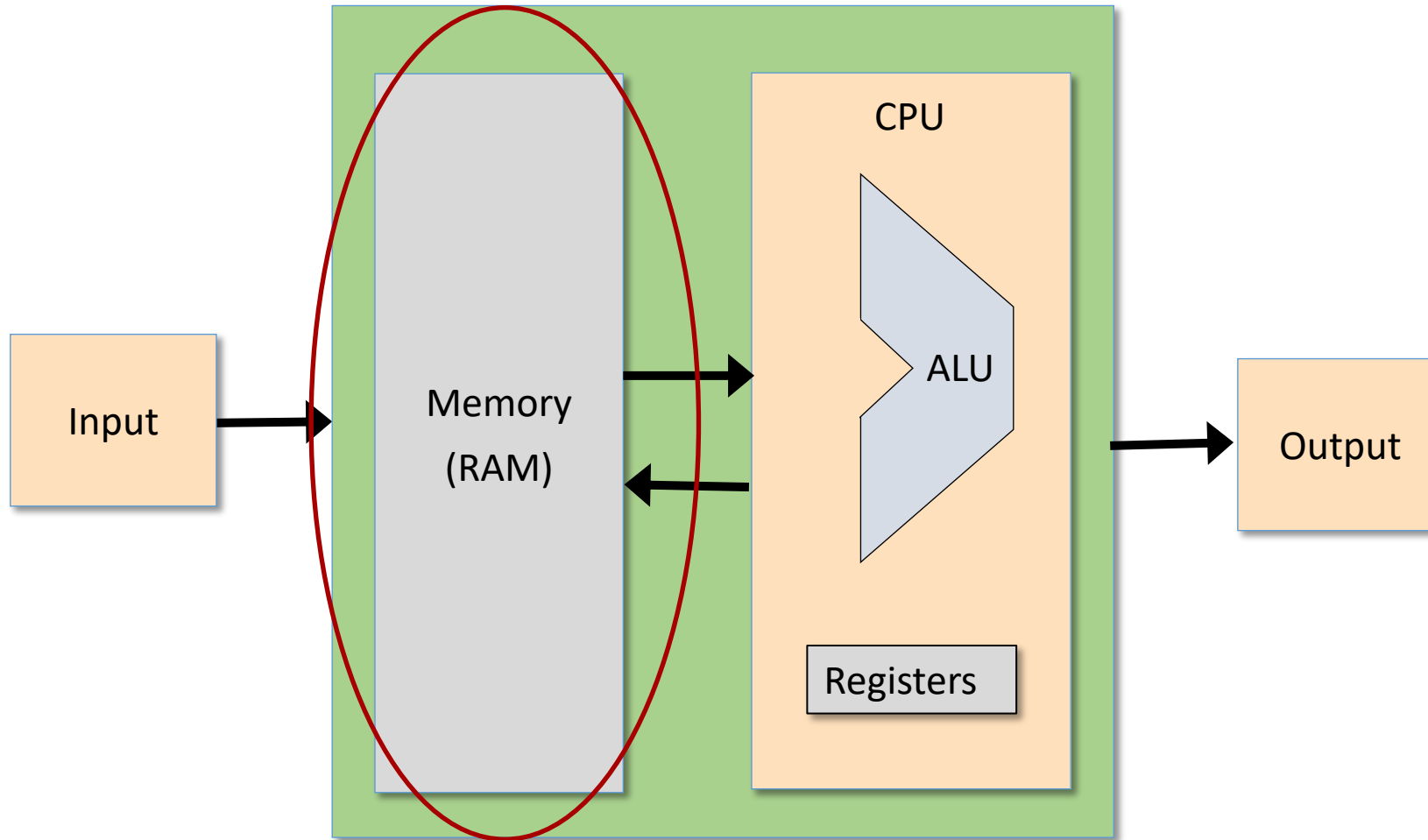
- Counters

## Implementation

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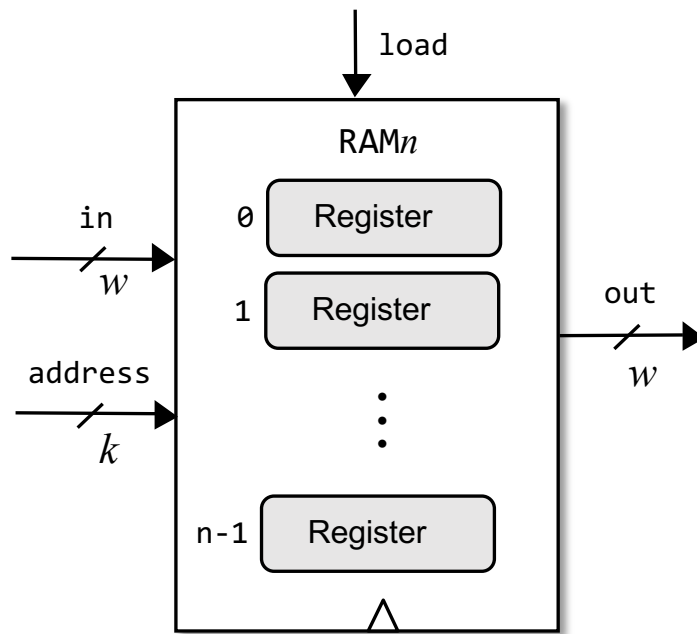
# Computer architecture

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# RAM

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## Practice question:

Suppose that the RAM size  $n = 8$  registers.

What should be the value of  $k$ ?

## Answer:

$$k = \log_2 n$$

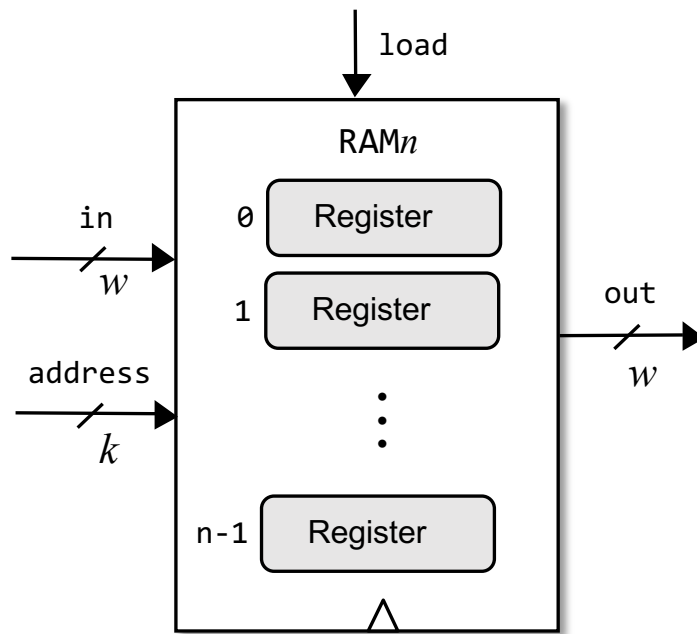
Abstraction: A sequence of  $n$  addressable,  $w$ -bit registers, with addresses 0 to  $n-1$

Word width: Typically 16, 32, 64 bits (Hack computer:  $w = 16$ )



# RAM

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## Behavior

If  $load == 0$ , the RAM maintains its state

If  $load == 1$ ,  $RAM[address]$  is set to the value of  $in$

The loaded value will be emitted by  $out$  from the next time-step (cycle) onward, until the next load

(Only one RAM register is selected;  
All the other registers are not affected)

## Usage:    **To read register $i$ :**

set  $address = i$ ,

probe  $out$  ( $out$  always emits the value of  $RAM[i]$ )

## **To write $v$ in register $i$ :**

set  $address = i$ ,

set  $in = v$ ,

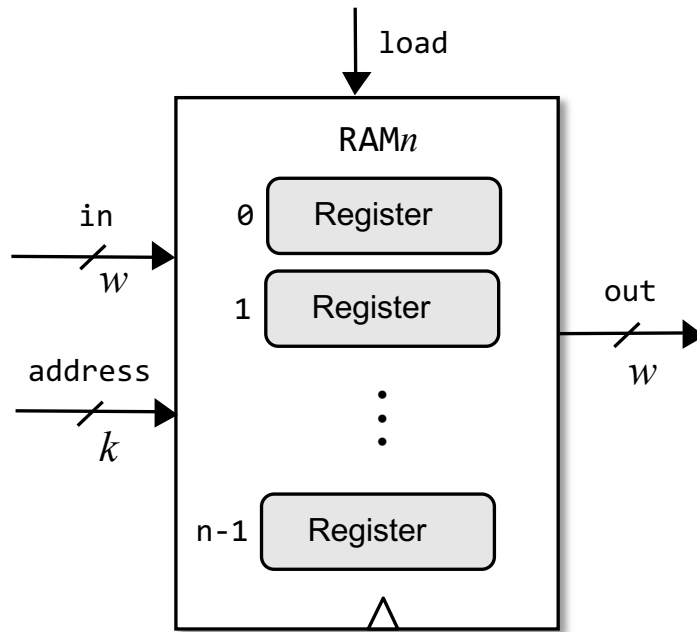
set  $load = 1$

Result:  $RAM[i] \leftarrow v$

From the next time-step onward,  $out$  will emit  $v$

# RAM

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## Why “Random Access Memory”?

Irrespective of the RAM size ( $n$ ), every randomly selected register can be accessed “instantaneously”, at more or less the same speed.



# Chapter 3: Memory

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## Abstraction

✓ Representing time

✓ Clock

✓ Registers

✓ RAM

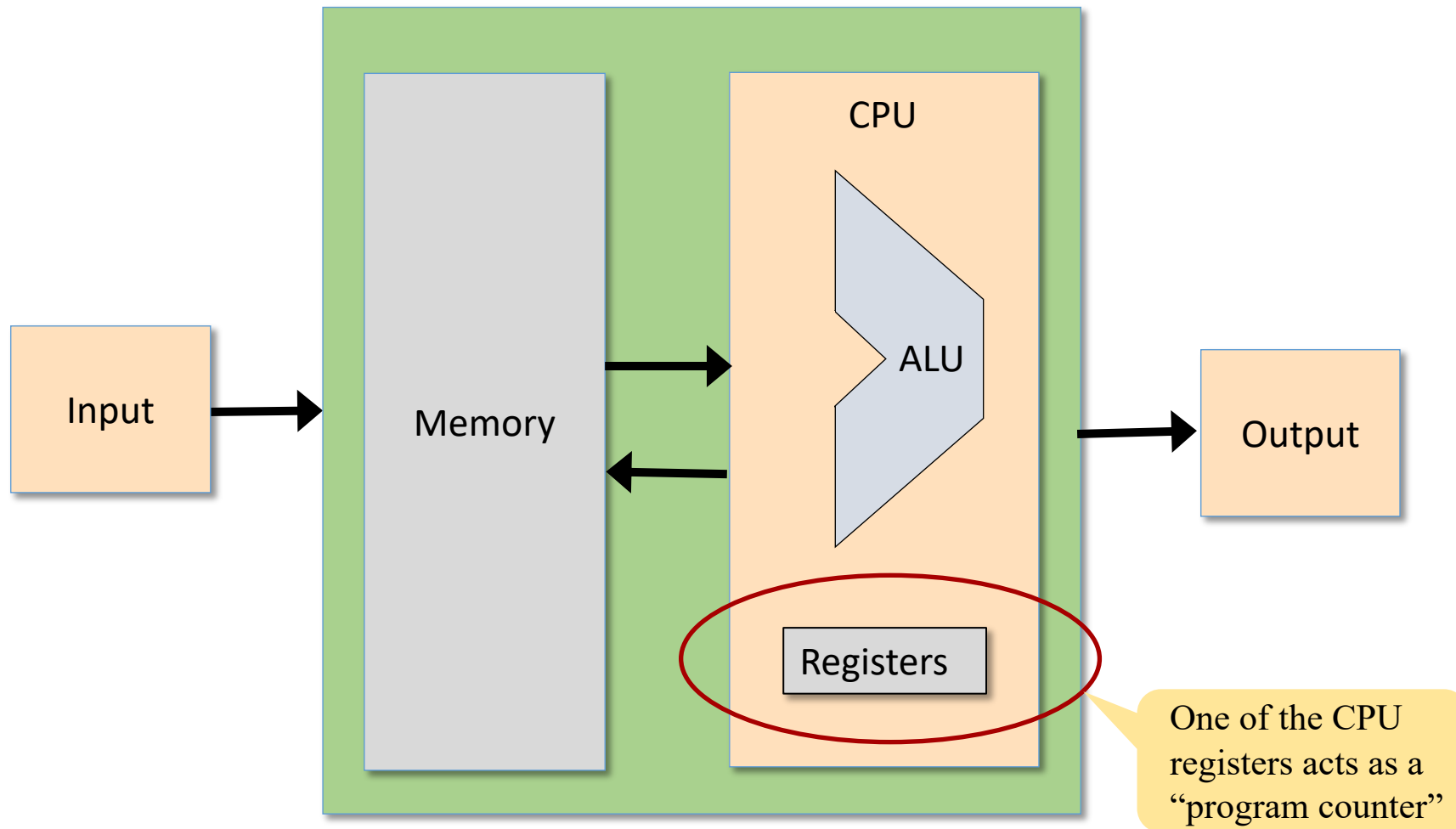
➡ Counters

## Implementation

- Data Flip Flop
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# Computer architecture

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# Counter

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- Later in the course (chapter 5), we will see that the computer must keep track of which instruction should be fetched and executed next
- This task is regulated by a register typically called Program Counter
- We'll use the PC to store the address of the instruction that should be fetched and executed next
- The PC should support three abstractions:

Reset: fetch the first instruction

PC = 0

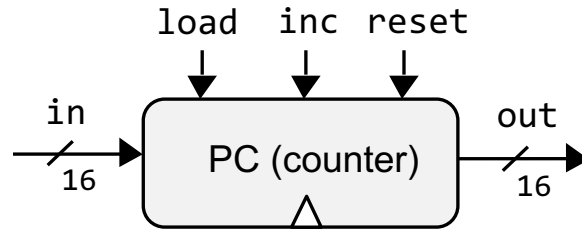
Next: fetch the next instruction

PC++

Goto: fetch instruction  $n$

PC =  $n$

# Counter



if $\text{reset}(t)$	$\text{out}(t+1) = 0$
else if $\text{load}(t)$	$\text{out}(t+1) = \text{in}(t)$
else if $\text{inc}(t)$	$\text{out}(t+1) = \text{out}(t) + 1$
else	$\text{out}(t+1) = \text{out}(t)$

## Usage:

### **To read:**

probe out

### **To set:**

set in to  $v$ ,  
assert load,  
set the other control bits to 0

### **To reset:**

assert reset,  
set the other control bits to 0

### **To count:**

assert inc,  
set the other control bits to 0



# Chapter 3: Memory

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## Abstraction

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# Chapter 3: Memory

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## Abstraction

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## Implementation



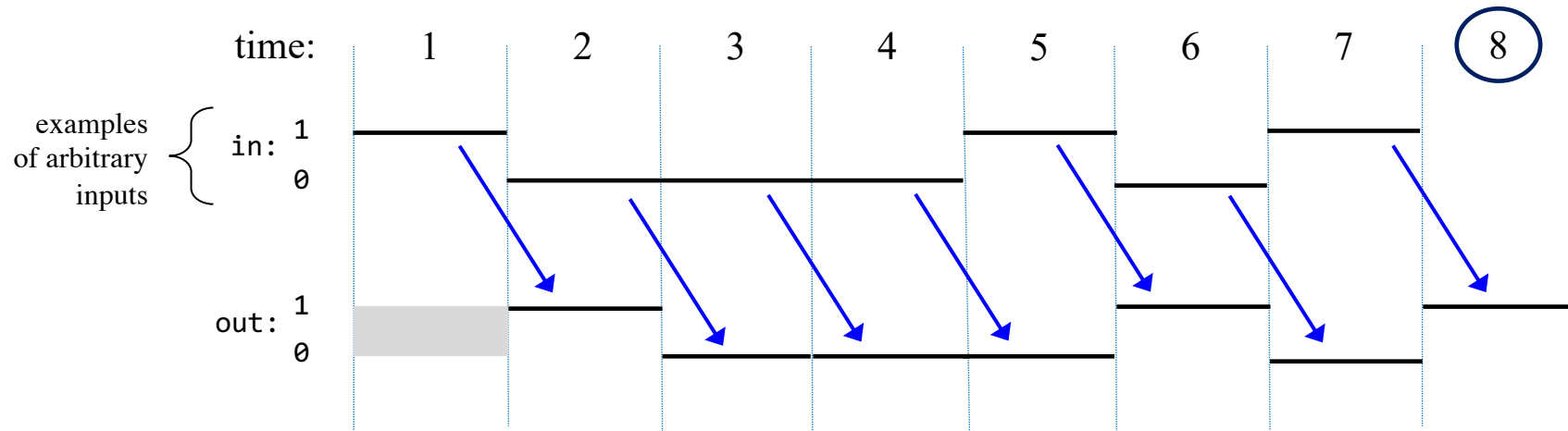
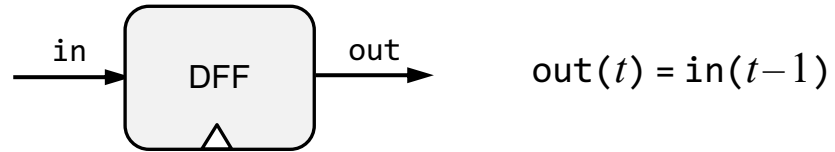
- Data Flip Flop
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- Project 3: Guidelines



# DFF

## Data Flip Flop (aka *latch*)

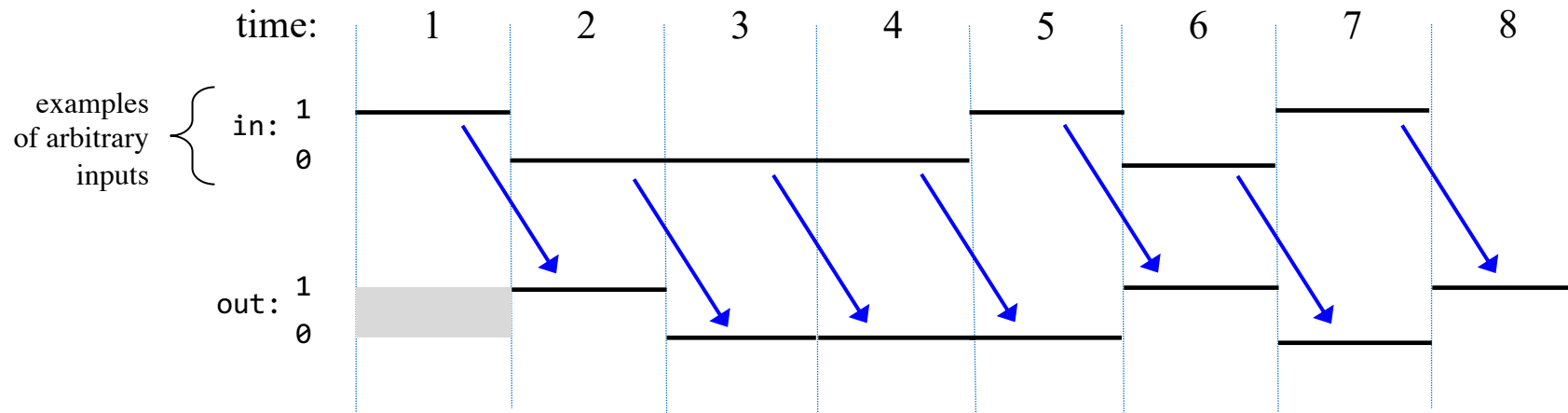
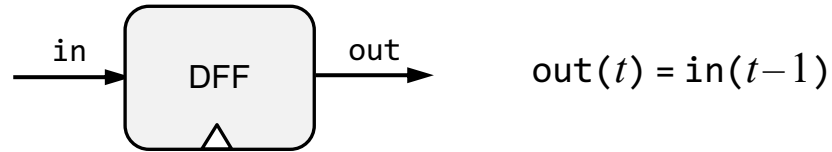
The most elementary sequential gate: Outputs the input in the previous time-step



# From DFF to a 1-bit register

## Data Flip Flop (aka *latch*)

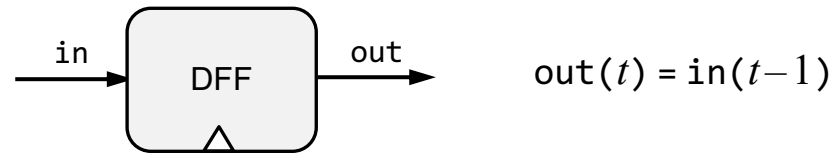
The most elementary sequential gate: Outputs the input in the previous time-step



How can we “load” and then “maintain” a value (0 or 1) over time, without having to feed the value in every cycle?

# From DFF to a 1-bit register

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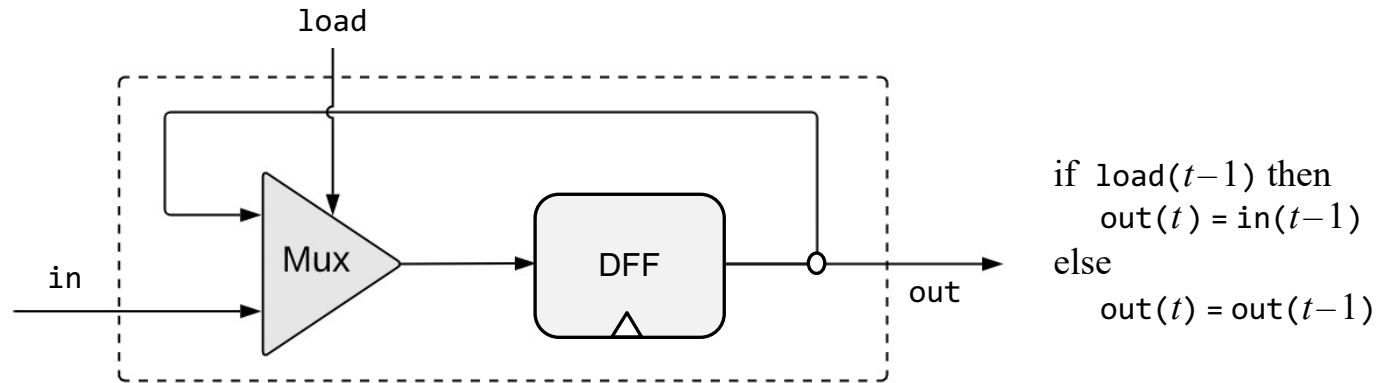
We have to realize a “loading” behavior and a ”storing” behavior, and be able to select between these two states

How can we “load” and then “maintain” a value (0 or 1) over time, without having to feed the value in every cycle?

# From DFF to a 1-bit register

## 1-bit Register

Stores one bit  
over time



if  $\text{load}(t-1)$  then  
     $\text{out}(t) = \text{in}(t-1)$   
else  
     $\text{out}(t) = \text{out}(t-1)$

We have to realize a “loading” behavior and a ”storing” behavior,  
and be able to select between these two states

## Behavior

if  $\text{load} == 1$  the register’s value becomes in  
else the register maintains its current value

# Register

---

## 1-bit Register

Stores one bit  
over time

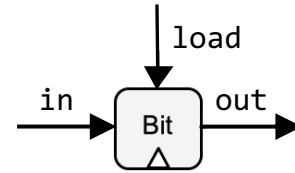


# Register

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## 1-bit Register

Stores one bit  
over time



zoom out...

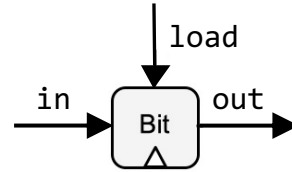
```
if load( $t-1$ ) then
    out( $t$ ) = in( $t-1$ )
else
    out( $t$ ) = out( $t-1$ )
```

# Register

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## 1-bit Register

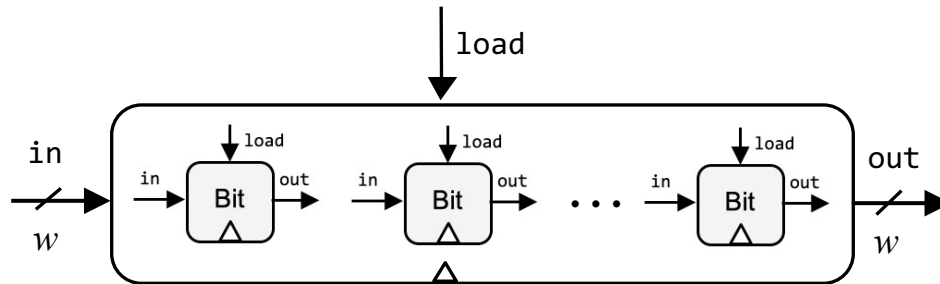
Stores one bit  
over time



if  $\text{load}(t-1)$  then  
     $\text{out}(t) = \text{in}(t-1)$   
else  
     $\text{out}(t) = \text{out}(t-1)$

## $w$ -bit Register:

Stores  $w$  bits  
over time



Partial diagram, showing  
some of the chip-parts,  
without connections




# Chapter 3: Memory

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## Abstraction

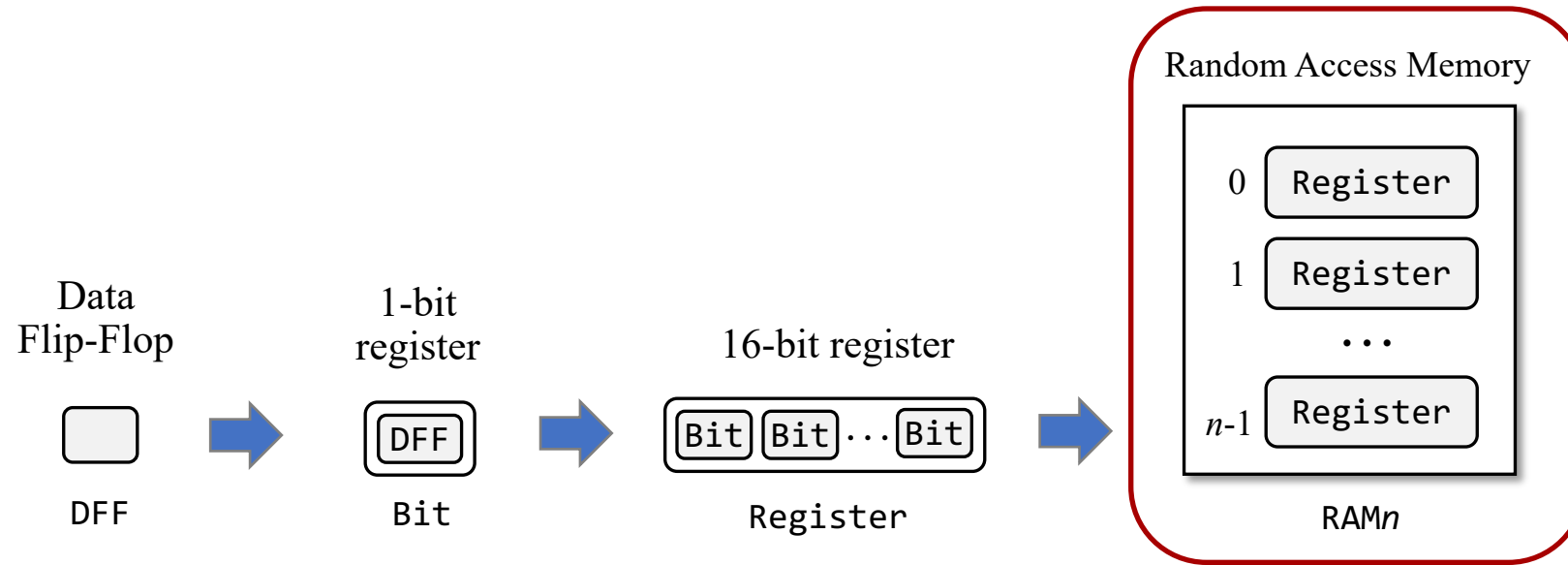
- Representing time
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## Implementation

-  Data Flip Flop
-  Registers
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  - Project 3: Chips
  - Project 3: Guidelines



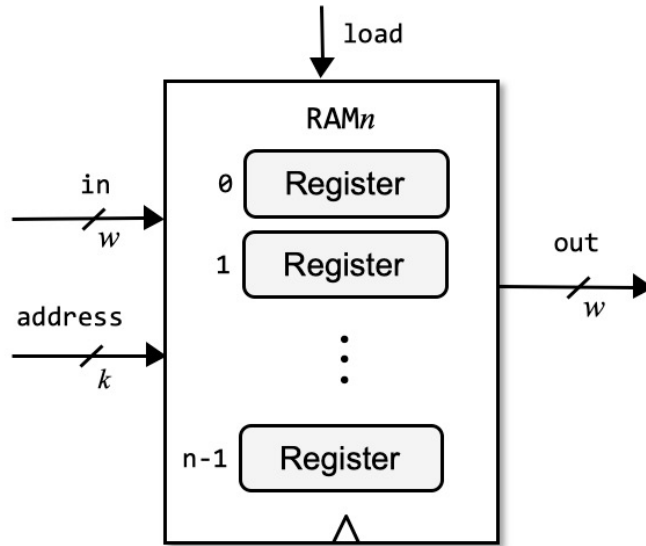
# Memory hierarchy



# RAM: Abstraction

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RAM of  $n$   
registers:

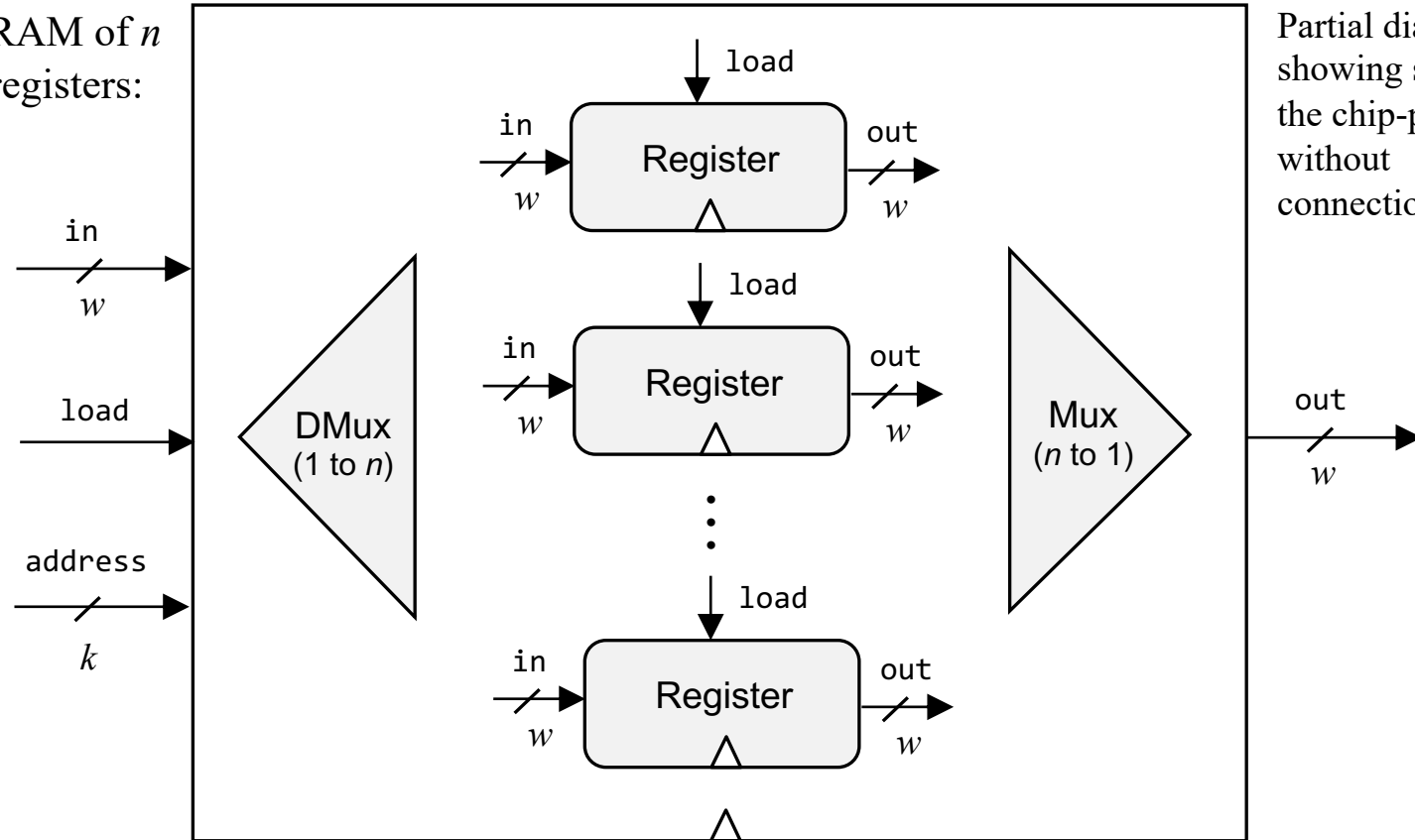


Usage:    **To read register  $i$  :**  
              set address =  $i$ ,  
              probe out (out always emits the state of  $RAM[i]$ )

**To write  $v$  in register  $i$  :**  
              set address =  $i$ ,  
              set in =  $v$ ,                    Result:  $RAM[i] \leftarrow v$   
              set load = 1                    From the next time-step onward, out emits  $v$

# RAM: Implementation

RAM of  $n$  registers:



Partial diagram,  
showing some of  
the chip-parts,  
without  
connections

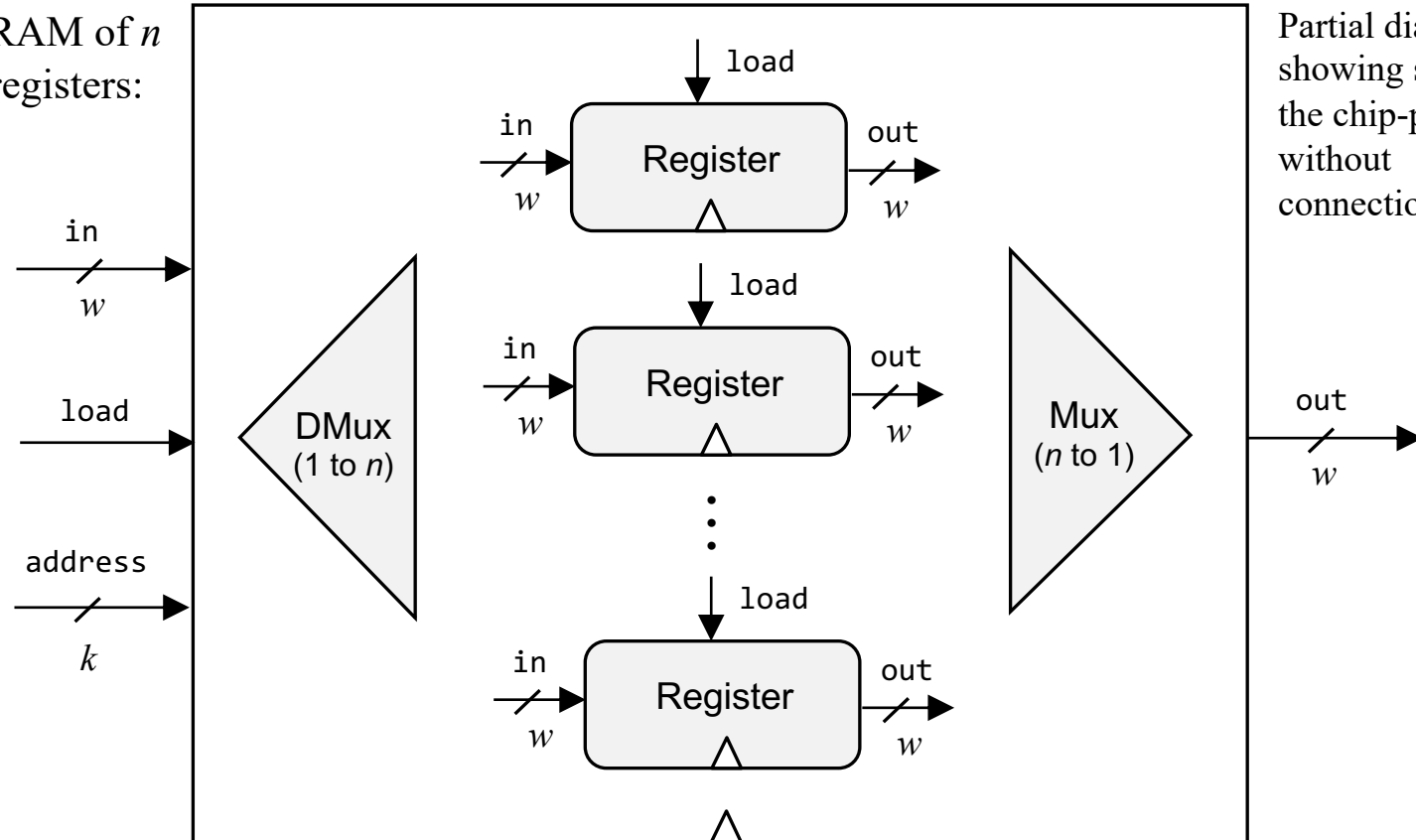
Reading: Can be realized using a Mux

Writing: Can be realized using a DMux

Connections?  
You figure it out

# RAM: Implementation

RAM of  $n$  registers:



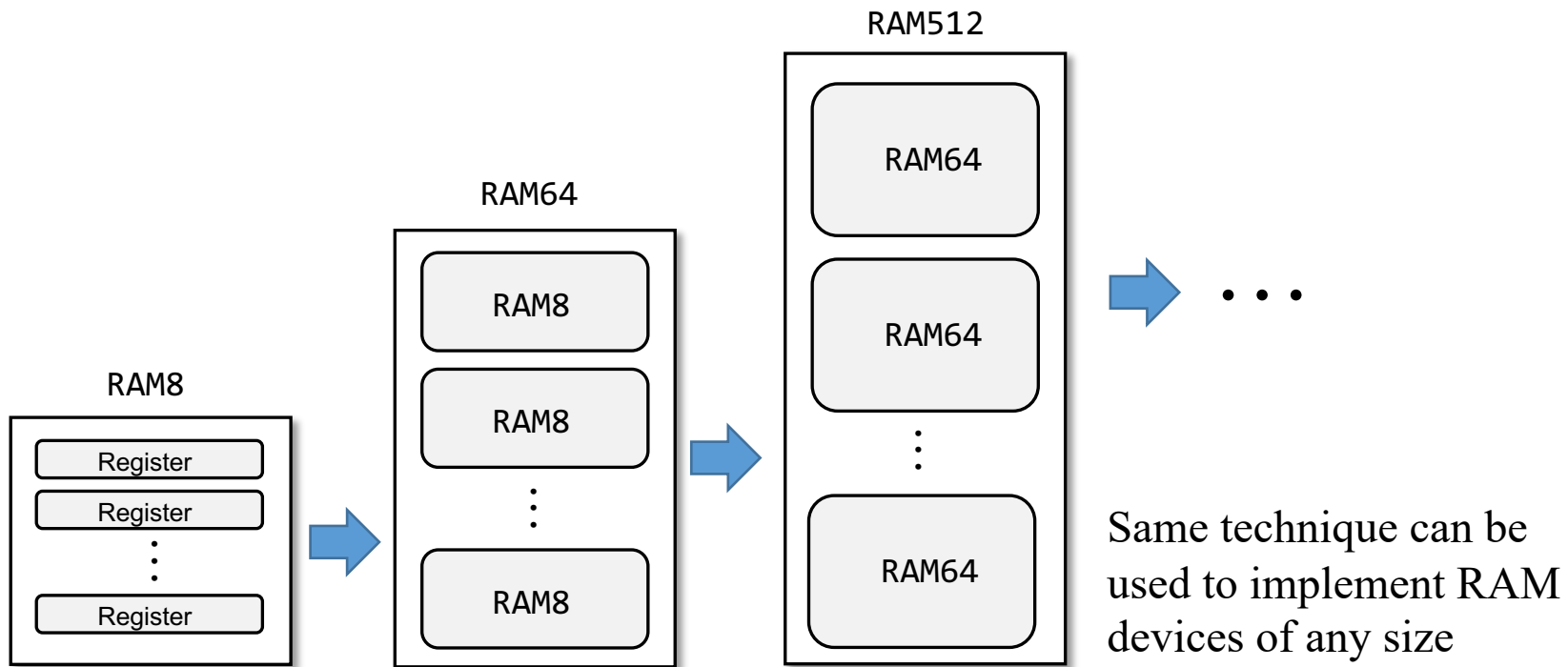
Partial diagram,  
showing some of  
the chip-parts,  
without  
connections

## Observations

- The addressing/selection/reading logic is *combinational*
- The writing logic is (i) *sequential (clocked)*  
(ii) *embedded in the Register logic.*

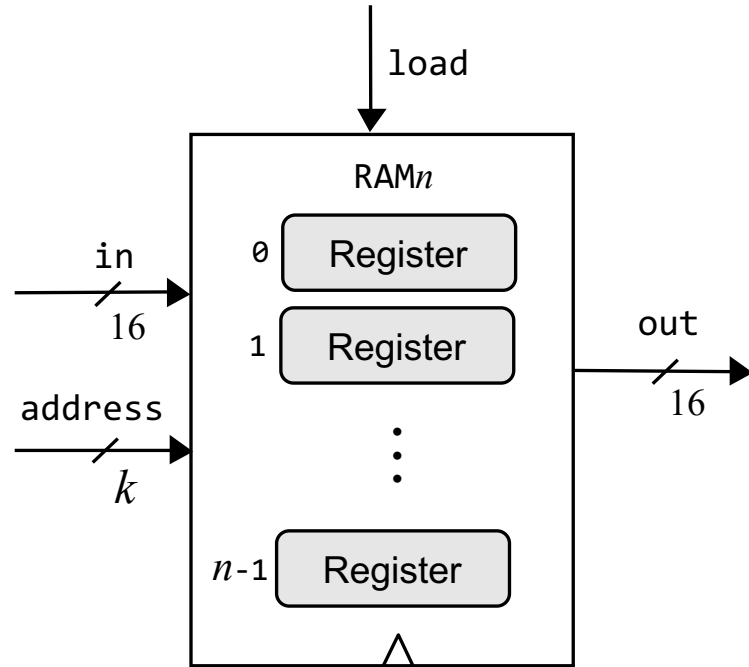
# RAM: Implementation

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# Hack RAM

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A family of 16-bit RAM chips:

chip name	$n$	$k$
RAM8	8	3
RAM64	64	6
RAM512	512	9
RAM4K	4096	12
RAM16K	16384	14

Why these particular RAM chips?

Because that's what we need for building the Hack computer.





# Chapter 3: Memory

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## Abstraction

- Representing time
- Clock
- Registers
- RAM
- Counters

## Implementation

-  Data Flip Flop
-  Registers
-  RAM
-  Project 3: Chips
  - Project 3: Guidelines

# Project 3

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## Given:

- All the chips built in projects 1 and 2
- Data Flip-Flop (built-in DFF gate)

## Build:

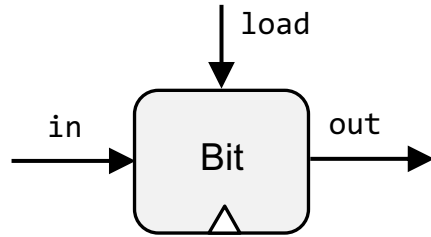


Bit

- Register
- PC
- RAM8
- RAM64
- RAM512
- RAM4K
- RAM16K



# 1-bit Register

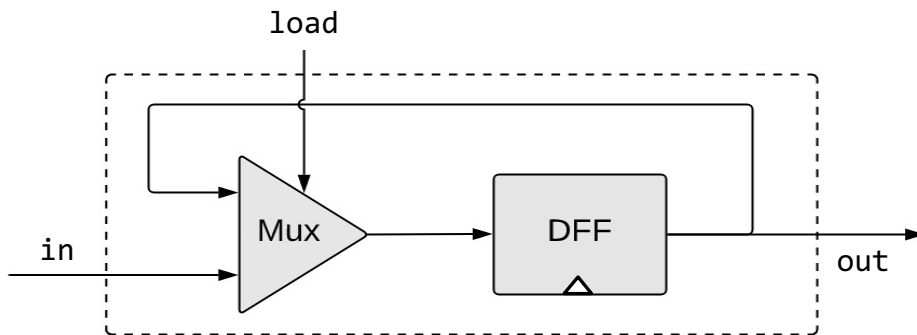


Bit.hdl

```
/** 1-bit register:
    if load(t-1) then out(t) = in(t-1)
    else
        out(t) = out(t-1)) */

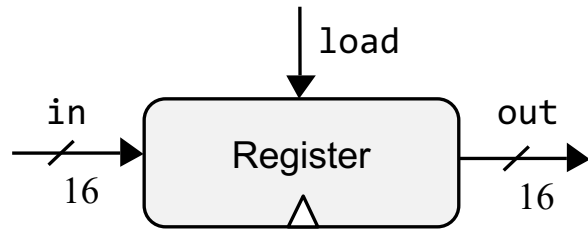
CHIP Bit {
    IN in, load;
    OUT out;

    PARTS:
    // Put your code here:
}
```



Implementation tip:  
Follow the chip diagram

# 16-bit Register

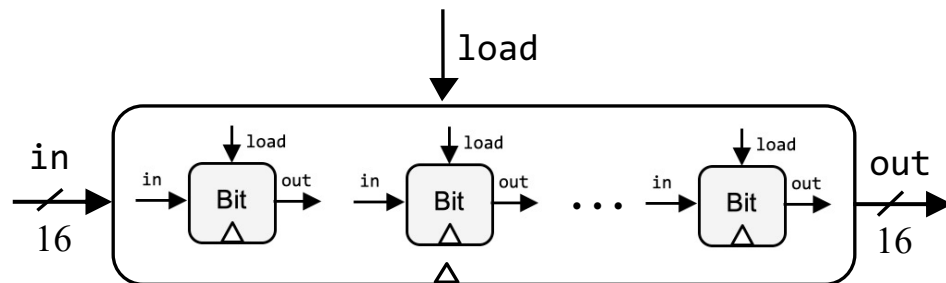


Register.hdl

```
/** 1-bit register:
    if load(t-1) then out(t) = in(t-1)
    else
        out(t) = out(t-1)) */

CHIP Bit {
    IN in[16], load;
    OUT out[16];

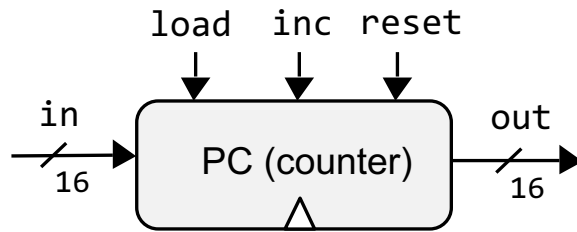
    PARTS:
    // Put your code here:
}
```



Partial diagram, showing some of  
the chip-parts, without connections

Implementation tip:  
Follow the chip diagram

# 16-bit Counter



```
/**
  A 16-bit counter with control bits.
  if      reset(t - 1) out(t) = 0           // resetting
  else if load(t - 1)  out(t) = in(t - 1)    // setting
  else if inc(t - 1)   out(t) = out(t - 1) + 1 // incrementing
  else                  out(t) = out(t - 1)    // maintaining
  */

CHIP PC {
  IN in[16], load, inc, reset;
  OUT out[16];

  PARTS:
    // Put your code here:
}
```

Implementation tip: Can be built from a Register, an Incrementer, and Mux's

# Project 3

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## Given

- All the chips built in projects 1 and 2
- Data Flip-Flop (built-in DFF gate)

## Build the following chips

✓ Bit

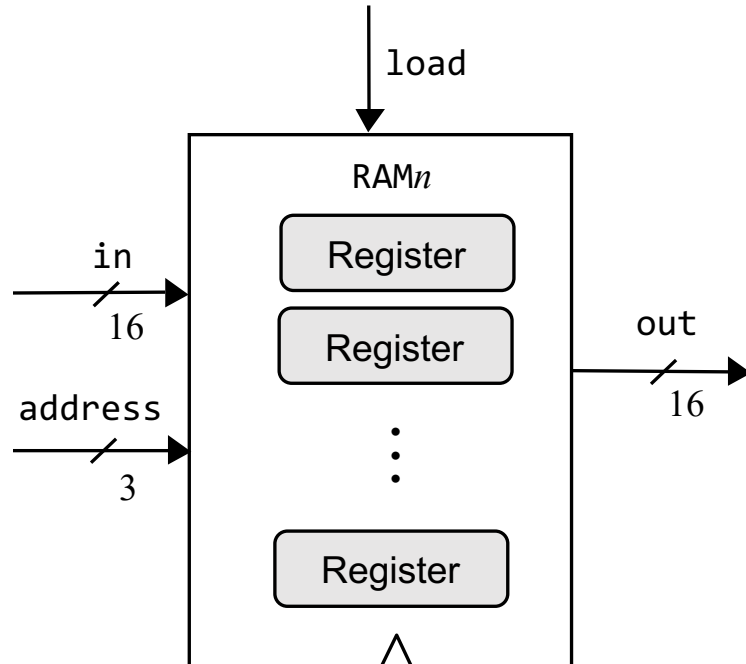
✓ Register

✓ PC

➔ RAM8

- RAM64
- RAM512
- RAM4K
- RAM16K

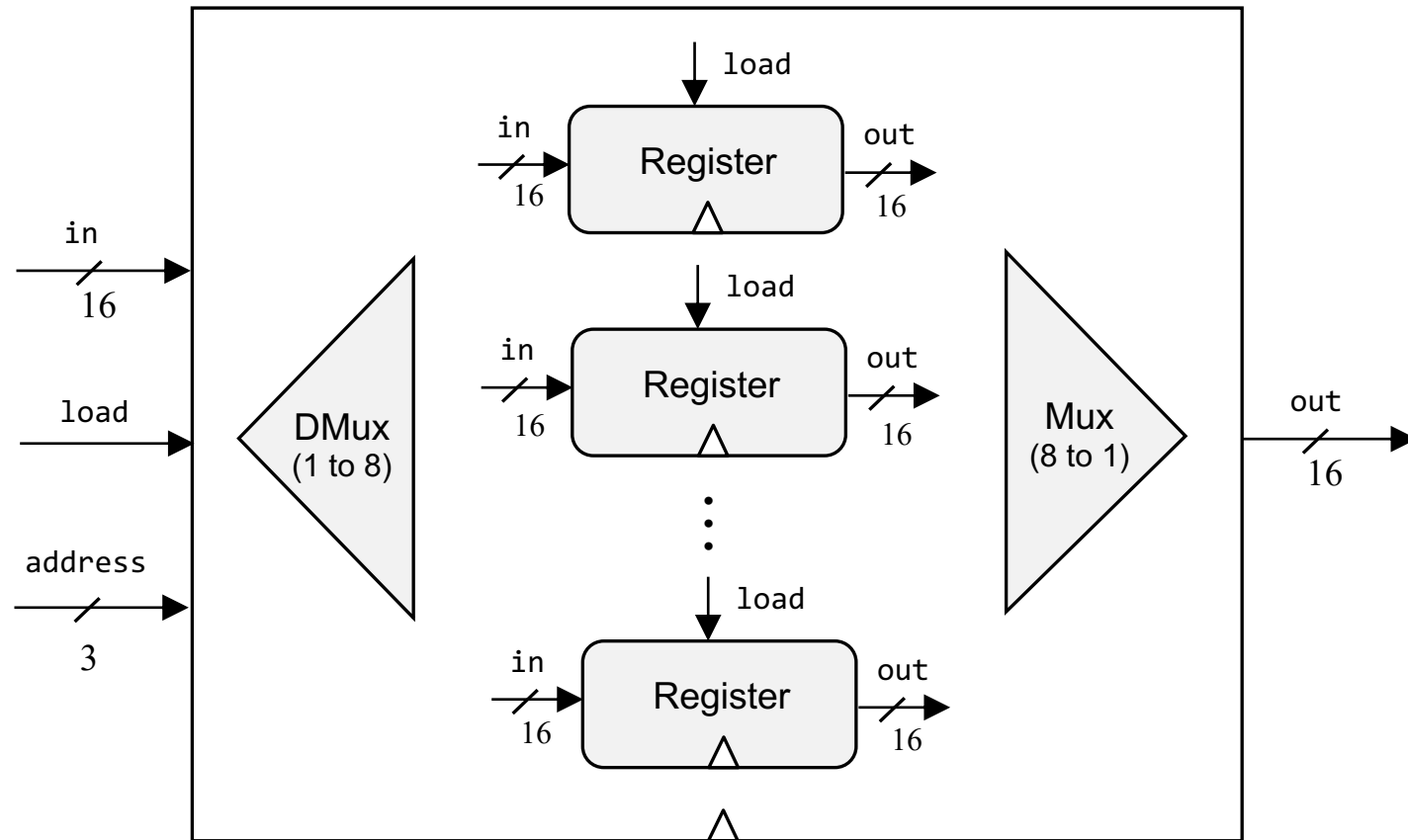
# 8-Register RAM: Abstraction



RAM8.hdl

```
/*
   Let M stand for the state of the register
   selected by address.
   if load(t - 1) then {M = in(t), out(t) = M}
   else
                           out(t) = M
*/
CHIP RAM8 {
    IN in[16], load, address[3];
    OUT out[16];
    PARTS:
        // Put your code here:
}
```

# 8-Register RAM: Implementation



Partial diagram, showing some of the chip-parts, without connections

Implementation tip:

Follow the chip diagram

# Project 3

---

## Given

- All the chips built in projects 1 and 2
- Data Flip-Flop (built-in DFF gate)

## Build the following chips

✓ Bit

✓ Register

✓ PC

✓ RAM8

• RAM64

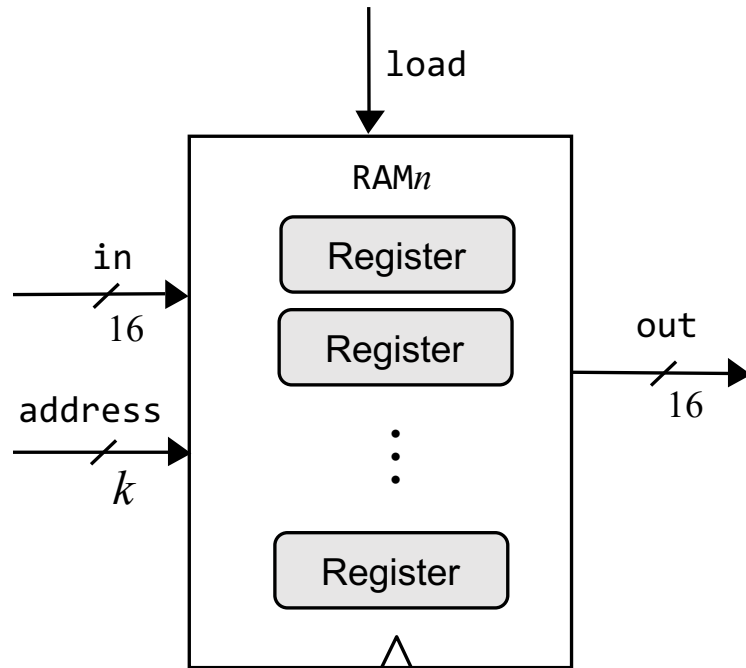
• RAM512

• RAM4K

• RAM16K

} A family of RAM chips

# $n$ -Register RAM

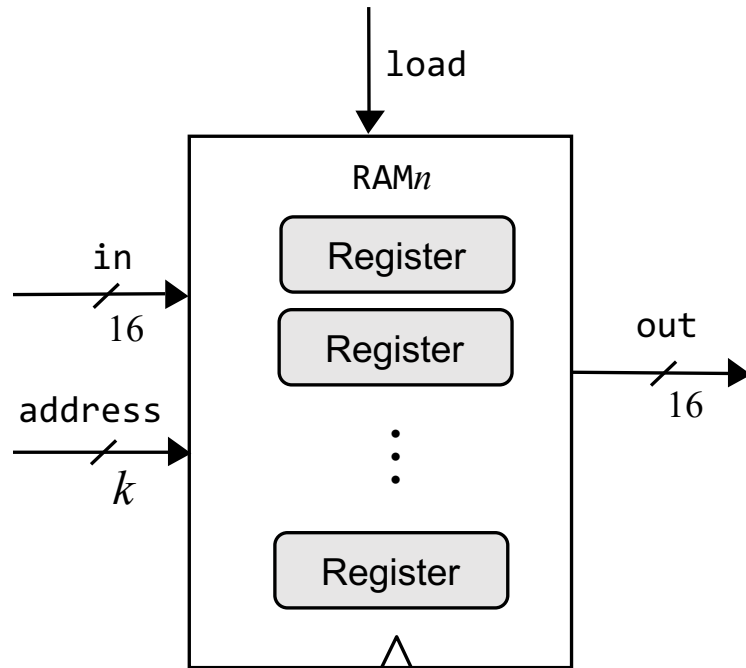


`RAMn.hdl`

```
/*
  Let M stand for the state of the register
  selected by address.
  if load(t - 1) then {M = in(t), out(t) = M}
  else
                        out(t) = M
*/
CHIP RAMn {
  IN in[16], load, address[k];
  OUT out[16];
  PARTS:
    // Put your code here:
}
```



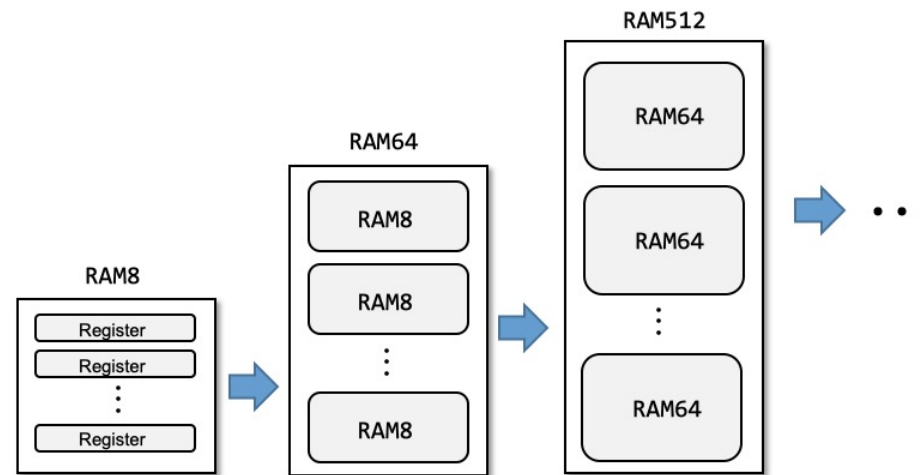
# $n$ -Register RAM



chip name	$n$	$k$
RAM8	8	3
RAM64	64	6
RAM512	512	9
RAM4K	4096	12
RAM16K	16384	14

## Implementation tips

- Think about the RAM's address input as consisting of two fields:
  - One field selects a RAM-part;
  - The other field selects a register within that RAM-part
- Use logic gates to effect this addressing scheme.








# Chapter 3: Memory

---

## Abstraction

- Representing time
- Clock
- Registers
- RAM
- Counters

## Implementation

-  Data Flip Flop
-  Registers
-  RAM
-  Project 3: Chips
-  Project 3: Guidelines

# Project 3

From NAND to Tetris  
Building a Modern Computer From First Principles

[www.nand2tetris.org](http://www.nand2tetris.org)



Home  
Prerequisites  
Syllabus

Course

Book  
Software  
Terms  
Papers  
Talks  
Cool Stuff  
About  
Team  
Q&A

## Project 3: Sequential Chips

### Background

The computer's main memory, also called *Random Access Memory*, or *RAM*, is an addressable sequence of  $n$ -bit registers, each designed to hold an  $n$ -bit value. In this project you will gradually build a RAM unit. This involves two main issues: (i) how to use gate logic to store bits persistently, over time, and (ii) how to use gate logic to locate ("address") the memory register on which we wish to operate.

### Objective

Build all the chips described in Chapter 3 (see list below), leading up to a *Random Access Memory* (RAM) unit. The only building blocks that you can use are primitive DFF gates, chips that you will build on top of them, and chips described in previous chapters.

### Chips

Chip (HDL)	Description	Test script	Compare file
DFF	Data Flip-Flop (primitive)		
Bit	1-bit register	<a href="#">Bit.tst</a>	<a href="#">Bit.cmp</a>
Register	16-bit register	<a href="#">Register.tst</a>	<a href="#">Register.cmp</a>
RAM8	16-bit / 8-register memory	<a href="#">RAM8.tst</a>	<a href="#">RAM8.cmp</a>
RAM64	16-bit / 64-register memory	<a href="#">RAM64.tst</a>	<a href="#">RAM64.cmp</a>
RAM512	16-bit / 512-register memory	<a href="#">RAM512.tst</a>	<a href="#">RAM512.cmp</a>
RAM4K	16-bit / 4096-register memory	<a href="#">RAM4K.tst</a>	<a href="#">RAM4K.cmp</a>
RAM16K	16-bit / 16384-register memory	<a href="#">RAM16K.tst</a>	<a href="#">RAM16K.cmp</a>
PC	16-bit program counter	<a href="#">PC.tst</a>	<a href="#">PC.cmp</a>

All the necessary project 3 files are available in:  
[nand2tetris / projects / 03](#)

# Resources

---

Project 3 folder (.hdl, .tst, .cmp files): `nand2tetris/projects/03`

## Tools

- Text editor (for completing the given .hdl stub-files)
- Hardware simulator: `nand2tetris/tools`

## Guides

- [Hardware Simulator Tutorial](#)
- [HDL Guide](#)
- [Hack Chip Set API](#)

# Best practice advice

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- Implement the chips in the order in which they appear in the project guidelines
- If you don't implement some chips, you can still use their built-in implementations
- No need for “helper chips”: Implement / use only the chips we specified
- In each chip definition, strive to use as few chip-parts as possible
- You will have to use chips implemented in previous projects;  
For efficiency and consistency's sake, use their built-in versions, rather than your own HDL implementations.

For technical reasons, the chips of project 3 are organized in two sub-folders named `projects/03/a` and `projects/03/b`

When writing and simulating the `.hdl` files, leave this folder structure as is.

That's It!  
Go Do Project 3!

# Chapter 3: Memory

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## Abstraction

- Representing time
- Clock
- Registers
- RAM
- Counters

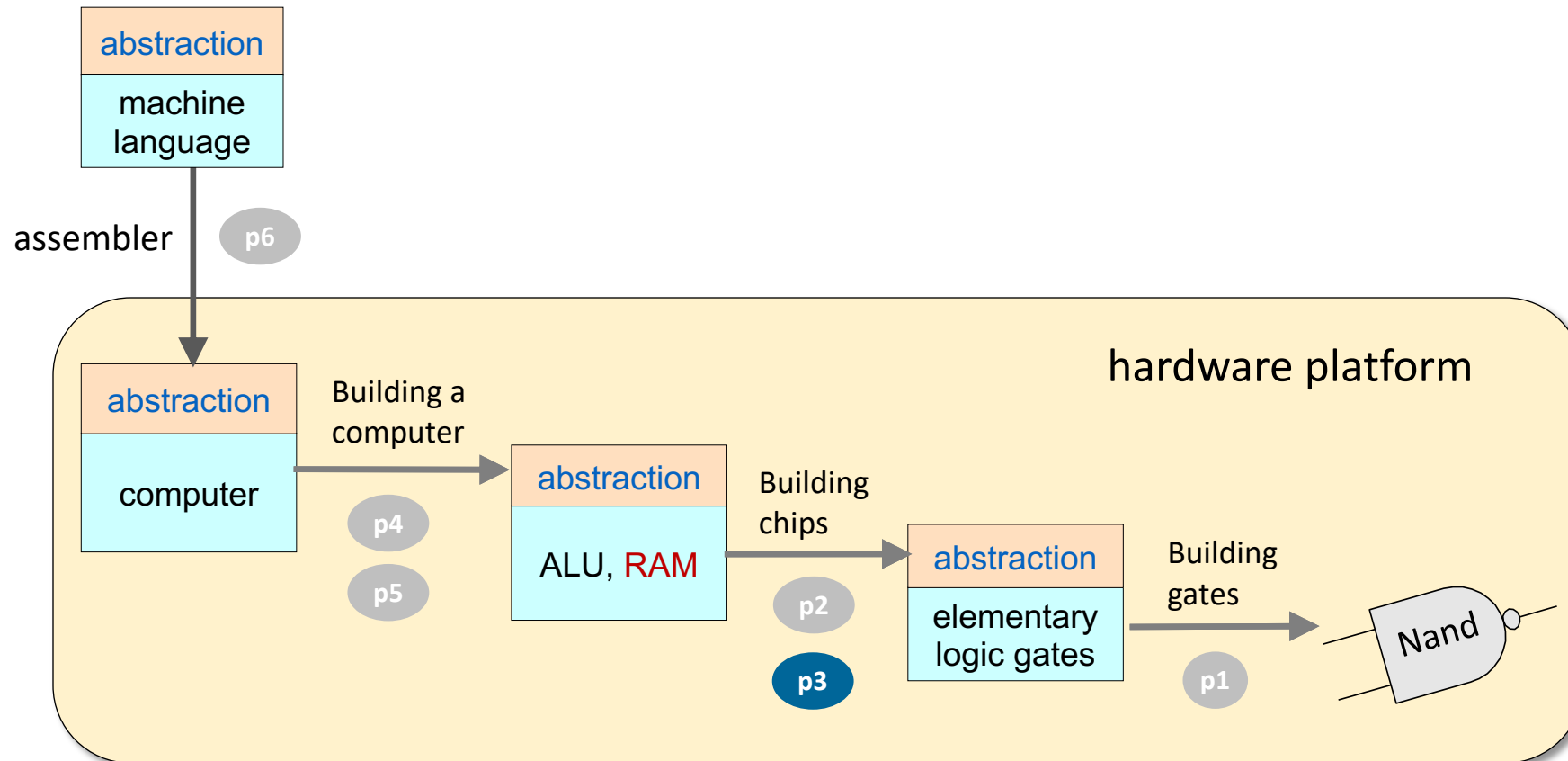


## Implementation

- Data Flip Flop
- Registers
- RAM
- Project 3: Chips
- Project 3: Guidelines

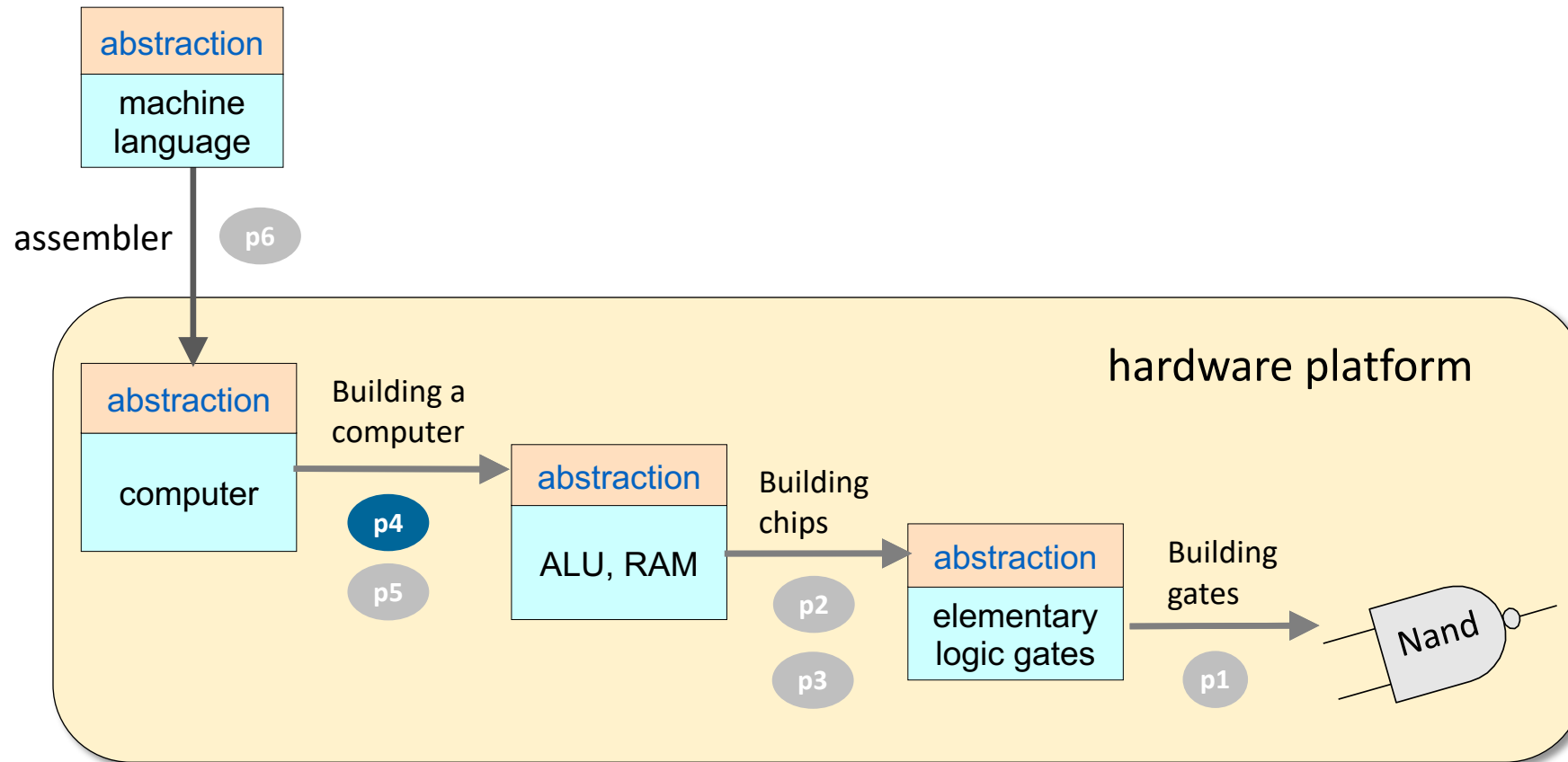


# What's next?



This lecture / chapter / project:  
Build the computer's RAM

# What's next?



Next lecture / chapter / project:

- Get acquainted with the computer architecture
- Write machine language programs